CentroAmerican 2001

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Day 1

1 Two players A, B and another 2001 people form a circle, such that A and B are not in consecutive positions. A and B play in alternating turns, starting with A. A play consists of touching one of the people neighboring you, which such person once touched leaves the circle. The winner is the last man standing.

Show that one of the two players has a winning strategy, and give such strategy.

Note: A player has a winning strategy if he/she is able to win no matter what the opponent does.

2 Let AB be the diameter of a circle with a center O and radius 1. Let C and D be two points on the circle such that AC and BD intersect at a point Q situated inside of the circle, and $\angle AQB = 2\angle COD$. Let P be a point that intersects the tangents to the circle that pass through the points C and D.

Determine the length of segment OP.

3 Find all the real numbers N that satisfy these requirements:

1. Only two of the digits of N are distinct from 0, and one of them is 3.

2. N is a perfect square.

Day 2

1 Determine the smallest positive integer n such that there exists positive integers $a_1, a_2, \cdots, a_n$, that smaller than or equal to 15 and are not necessarily distinct, such that the last four digits of the sum,

$$a_1! + a_2! + \cdots + a_n!$$

Is 2001.

2 Let a, b and c real numbers such that the equation $ax^2+bx+c=0$ has two distinct real solutions p_1, p_2 and the equation $cx^2+bx+a=0$ has two distinct real solutions q_1, q_2. We know that the numbers p_1, q_1, p_2, q_2 in that order, form an arithmetic progression. Show that $a + c = 0$.

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3 In a circumference of a circle, 10000 points are marked, and they are numbered from 1 to 10000 in a clockwise manner. 5000 segments are drawn in such a way so that the following conditions are met:

1. Each segment joins two marked points.
2. Each marked point belongs to one and only one segment.
3. Each segment intersects exactly one of the remaining segments.
4. A number is assigned to each segment that is the product of the number assigned to each end point of the segment.

Let S be the sum of the products assigned to all the segments.

Show that S is a multiple of 4.
