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Day 1 July 2nd

1 For what integers $n \geq 3$ is it possible to accommodate, in some order, the numbers $1, 2, \cdots, n$ in a circular form such that every number divides the sum of the next two numbers, in a clockwise direction?

2 Let ABC be an acute triangle, and let D and E be the feet of the altitudes drawn from vertexes A and B, respectively. Show that if,

$$Area[BDE] \leq Area[DEA] \leq Area[EAB] \leq Area[ABD]$$

then, the triangle is isosceles.

3 For every integer $a > 1$ an infinite list of integers is constructed $L(a)$, as follows:

a is the first number in the list $L(a)$.
Given a number b in $L(a)$, the next number in the list is $b + c$, where c is the largest integer that divides b and is smaller than b.
Find all the integers $a > 1$ such that 2002 is in the list $L(a)$.

Day 2 July 3rd

4 Let ABC be a triangle, D be the midpoint of BC, E be a point on segment AC such that $BE = 2AD$ and F is the intersection point of AD with BE. If $\angle DAC = 60^\circ$, find the measure of the angle FEA.

5 Find a set of infinite positive integers S such that for every $n \geq 1$ and whichever n distinct elements $x_1, x_2, \cdots, x_n$ of S, the number $x_1 + x_2 + \cdots + x_n$ is not a perfect square.

6 A path from $(0, 0)$ to (n, n) on the lattice is made up of unit moves upward or rightward. It is balanced if the sum of the x-coordinates of its $2n + 1$ vertices equals the sum of their y-coordinates. Show that a balanced path divides the square with vertices $(0, 0)$, $(n, 0)$, (n, n), $(0, n)$ into two parts with equal area.