**CentroAmerican 2005**

www.artofproblemsolving.com/community/c4561

by Jutaro

**Day 1** June 21st

---

**1** Among the positive integers that can be expressed as the sum of 2005 consecutive integers, which occupies the 2005th position when arranged in order?

*Roland Hablutzel, Venezuela*

The original question was: Among the positive integers that can be expressed as the sum of 2004 consecutive integers, and also as the sum of 2005 consecutive integers, which occupies the 2005th position when arranged in order?

---

**2** Show that the equation $a^2b^2 + b^2c^2 + 3b^2 - c^2 - a^2 = 2005$ has no integer solutions.

*Arnoldo Aguilar, El Salvador*

---

**3** Let $ABC$ be a triangle. $P$, $Q$ and $R$ are the points of contact of the incircle with sides $AB$, $BC$ and $CA$, respectively. Let $L$, $M$ and $N$ be the feet of the altitudes of the triangle $PQR$ from $R$, $P$ and $Q$, respectively.

a) Show that the lines $AN$, $BL$ and $CM$ meet at a point.

b) Prove that this points belongs to the line joining the orthocenter and the circumcenter of triangle $PQR$.

*Aarn Ramrez, El Salvador*

---

**Day 2** June 22nd

---

**4** Two players, Red and Blue, play in alternating turns on a 10x10 board. Blue goes first. In his turn, a player picks a row or column (not chosen by any player yet) and color all its squares with his own color. If any of these squares was already colored, the new color substitutes the old one.

The game ends after 20 turns, when all rows and column were chosen. Red wins if the number of red squares in the board exceeds at least by 10 the number of blue squares; otherwise Blue wins.

Determine which player has a winning strategy and describe this strategy.

**5** Let $ABC$ be a triangle, $H$ the orthocenter and $M$ the midpoint of $AC$. Let $\ell$ be the parallel through $M$ to the bisector of $\angle AHC$. Prove that $\ell$ divides the triangle in two parts of equal perimeters.

*Pedro Marrone, Panam*

**6** Let $n$ be a positive integer and $p$ a fixed prime. We have a deck of $n$ cards, numbered $1,\ 2, \ldots,\ n$ and $p$ boxes for put the cards on them. Determine all posible integers $n$ for which is possible to distribute the cards in the boxes in such a way the sum of the numbers of the cards in each box is the same.