CentroAmerican 2007

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Day 1 June 5th

1 The Central American Olympiad is an annual competition. The ninth Olympiad is held in 2007. Find all the positive integers n such that n divides the number of the year in which the n-th Olympiad takes place.

2 In a triangle ABC, the angle bisector of A and the cevians BD and CE concur at a point P inside the triangle. Show that the quadrilateral $ADPE$ has an incircle if and only if $AB = AC$.

3 Let S be a finite set of integers. Suppose that for every two different elements of S, p and q, there exist not necessarily distinct integers $a \neq 0$, b, c belonging to S, such that p and q are the roots of the polynomial $ax^2 + bx + c$. Determine the maximum number of elements that S can have.

Day 2 June 6th

1 In a remote island, a language in which every word can be written using only the letters a, b, c, d, e, f, g is spoken. Let's say two words are *synonymous* if we can transform one into the other according to the following rules:

i) Change a letter by another two in the following way:

$$a \to bc,\ b \to cd,\ c \to de,\ d \to ef,\ e \to fg,\ f \to ga,\ g \to ab$$

ii) If a letter is between other two equal letters, these can be removed. For example, $dfd \to f$.

Show that all words in this language are synonymous.

2 Given two non-negative integers $m > n$, let's say that m *ends in* n if we can get n by erasing some digits (from left to right) in the decimal representation of m. For example, 329 ends in 29, and also in 9.

Determine how many three-digit numbers end in the product of their digits.

3 Consider a circle S, and a point P outside it. The tangent lines from P meet S at A and B, respectively. Let M be the midpoint of AB. The perpendicular bisector of AM meets S in a point C lying inside the triangle ABP. AC intersects PM at G, and PM meets S in a point D lying outside the triangle ABP. If BD is parallel to AC, show that G is the centroid of the triangle

ABP.

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