CentroAmerican 2008

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1 Find the least positive integer N such that the sum of its digits is 100 and the sum of the digits of $2N$ is 110.

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2 Let $ABCD$ be a convex cuadrilateral inscribed in a circumference centered at O such that AC is a diameter. Pararellograms $DAOE$ and $BCOF$ are constructed. Show that if E and F lie on the circumference then $ABCD$ is a rectangle.

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3 There are 2008 bags numbered from 1 to 2008, with 2008 frogs in each one of them. Two people play in turns. A play consists in selecting a bag and taking out of it any number of frongs (at least one), leaving x frogs in it ($x \geq 0$). After each play, from each bag with a number higher than the selected one and having more than x frogs, some frogs scape until there are x frogs in the bag. The player that takes out the last frog from bag number 1 looses. Find and explain a winning strategy.

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4 Five girls have a little store that opens from Monday through Friday. Since two people are always enough for taking care of it, they decide to do a work plan for the week, specifying who will work each day, and fulfilling the following conditions:

a) Each girl will work exactly two days a week
b) The 5 assigned couples for the week must be different
In how many ways can the girls do the work plan?

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5 Find a polynomial $p(x)$ with real coefficients such that
$(x+10)\,p(2x) = (8x-32)\,p(x+6)$

for all real x and $p(1) = 210$.

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6 Let ABC be an acute triangle. Take points P and Q inside AB and AC, respectively, such that $BPQC$ is cyclic. The circumcircle of ABQ intersects BC again in S and the circumcircle of APC intersects BC again in R, PR and QS intersect again in L. Prove that the intersection of AL and BC does not depend on the selection of P and Q.
