CentroAmerican 2009

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by Concyclicboy

Day 1 October 6th

1 Let P be the product of all non-zero digits of the positive integer n. For example, $P(4) = 4$, $P(50) = 5$, $P(123) = 6$, $P(2009) = 18$.
Find the value of the sum: P(1) + P(2) + ... + P(2008) + P(2009).

2

Two circles Γ_1 and Γ_2 intersect at points A and B. Consider a circle Γ contained in Γ_1 and Γ_2, which is tangent to both of them at D and E respectively. Let C be one of the intersection points of line AB with Γ, F be the intersection of line EC with Γ_2 and G be the intersection of line DC with Γ_1. Let H and I be the intersection points of line ED with Γ_1 and Γ_2 respectively. Prove that F, G, H and I are on the same circle.

3 There are 2009 boxes numbered from 1 to 2009, some of which contain stones. Two players, A and B, play alternately, starting with A. A move consists in selecting a non-empty box i, taking one or more stones from that box and putting them in box $i+1$. If $i = 2009$, the selected stones are eliminated. The player who removes the last stone wins
a) If there are 2009 stones in the box 2 and the others are empty, find a winning strategy for either player.
b) If there is exactly one stone in each box, find a winning strategy for either player.

Day 2 October 7th

4 We wish to place natural numbers around a circle such that the following property is satisfied: the absolute values of the differences of each pair of neighboring numbers are all different.
a) Is it possible to place the numbers from 1 to 2009 satisfying this property
b) Is it possible to suppress one of the numbers from 1 to 2009 in such a way that the remaining 2008 numbers can be placed satisfying the property

5 Given an acute and scalene triangle ABC, let H be its orthocenter, O its circumcenter, E and F the feet of the altitudes drawn from B and C, respectively. Line AO intersects the circumcircle of the triangle again at point G and segments FE and BC at points X and Y respectively. Let Z be the point of intersection of line AH and the tangent line to the circumcircle at G. Prove that HX is parallel to YZ.

6 Find all prime numbers p and q such that $p^3 - q^5 = (p+q)^2$.