

**CentroAmerican 2014**

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**Day 1** June 10th

**1** A positive integer is called *tico* if it is the product of three different prime numbers that add up to 74. Verify that 2014 is tico. Which year will be the next tico year? Which one will be the last tico year in history?

**2** Let  $ABCD$  be a trapezoid with bases  $AB$  and  $CD$ , inscribed in a circle of center  $O$ . Let  $P$  be the intersection of the lines  $BC$  and  $AD$ . A circle through  $O$  and  $P$  intersects the segments  $BC$  and  $AD$  at interior points  $F$  and  $G$ , respectively. Show that  $BF = DG$ .

**3** Let  $a, b, c$  and  $d$  be real numbers such that no two of them are equal,

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} = 4$$

and  $ac = bd$ . Find the maximum possible value of

$$\frac{a}{c} + \frac{b}{d} + \frac{c}{a} + \frac{d}{b}.$$

**Day 2** June 11th

**1** Using squares of side 1, a stair-like figure is formed in stages following the pattern of the drawing.

For example, the first stage uses 1 square, the second uses 5, etc. Determine the last stage for which the corresponding figure uses less than 2014 squares.

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**2** Points  $A, B, C$  and  $D$  are chosen on a line in that order, with  $AB$  and  $CD$  greater than  $BC$ . Equilateral triangles  $APB, BCQ$  and  $CDR$  are constructed so that  $P, Q$  and  $R$  are on the same side with respect to  $AD$ . If  $\angle PQR = 120^\circ$ , show that

$$\frac{1}{AB} + \frac{1}{CD} = \frac{1}{BC}.$$

- 3 A positive integer  $n$  is *funny* if for all positive divisors  $d$  of  $n$ ,  $d + 2$  is a prime number. Find all funny numbers with the largest possible number of divisors.
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