Iranian Geometry Olympiad 2016

www.artofproblemsolving.com/community/c457380

by Snakes, parmenides51, cjquines0, MRF2017

– Elementary

1 Ali wants to move from point A to point B. He cannot walk inside the black areas but he is free to move in any direction inside the white areas (not only the grid lines but the whole plane). Help Ali to find the shortest path between A and B. Only draw the path and write its length.
`https://1.bp.blogspot.com/-nZrxJLfIAp8/W1RyCdnhl3I/AAAAAAAAIzQ/NM3t5EtJWMcWQS0ig0IghSo54I s1600/igo%2B2016.el1.png`
by Morteza Saghafian

2 Let ω be the circumcircle of triangle ABC with $AC > AB$. Let X be a point on AC and Y be a point on the circle ω, such that $CX = CY = AB$. (The points A and Y lie on different sides of the line BC). The line XY intersects ω for the second time in point P. Show that $PB = PC$.

by Iman Maghsoudi

3 Suppose that $ABCD$ is a convex quadrilateral with no parallel sides. Make a parallelogram on each two consecutive sides. Show that among these 4 new points, there is only one point inside the quadrilateral $ABCD$.

by Morteza Saghafian

4 In a right-angled triangle ABC ($\angle A = 90^o$), the perpendicular bisector of BC intersects the line AC in K and the perpendicular bisector of BK intersects the line AB in L. If the line CL be the internal bisector of angle C, find all possible values for angles B and C.

by Mahdi Etesami Fard

5 Let $ABCD$ be a convex quadrilateral with these properties: $\angle ADC = 135^o$ and $\angle ADB - \angle ABD = 2\angle DAB = 4\angle CBD$.
If $BC = \sqrt{2}CD$, prove that $AB = BC + AD$.

by Mahdi Etesami Fard

– Medium

1 In trapezoid $ABCD$ with $AB||CD$, ω_1 and ω_2 are two circles with diameters AD and BC, respectively. Let X and Y be two arbitrary points on ω_1 and ω_2, respectively. Show that the length of segment XY is not more than half the perimeter of $ABCD$.

Proposed by Mahdi Etesami Fard

2 Let two circles C_1 and C_2 intersect in points A and B. The tangent to C_1 at A intersects C_2 in P and the line PB intersects C_1 for the second time in Q (suppose that Q is outside C_2). The tangent to C_2 from Q intersects C_1 and C_2 in C and D, respectively. (The points A and D lie on different sides of the line PQ.) Show that AD is the bisector of $\angle CAP$.

Proposed by Iman Maghsoudi

3 Find all positive integers N such that there exists a triangle which can be dissected into N similar quadrilaterals.

Proposed by Nikolai Beluhov (Bulgaria) and Morteza Saghafian

4 Let ω be the circumcircle of right-angled triangle ABC ($\angle A = 90^\circ$). The tangent to ω at point A intersects the line BC at point P. Suppose that M is the midpoint of the minor arc AB, and PM intersects ω for the second time in Q. The tangent to ω at point Q intersects AC at K. Prove that $\angle PKC = 90^\circ$.

Proposed by Davood Vakili

5 Let the circles ω and ω' intersect in points A and B. The tangent to circle ω at A intersects ω' at C and the tangent to circle ω' at A intersects ω at D. Suppose that the internal bisector of $\angle CAD$ intersects ω and ω' at E and F, respectively, and the external bisector of $\angle CAD$ intersects ω and ω' at X and Y, respectively. Prove that the perpendicular bisector of XY is tangent to the circumcircle of triangle BEF.

Proposed by Mahdi Etesami Fard

– Advanced

1 Let the circles ω and ω' intersect in A and B. Tangent to circleω at A intersectsω' in C and tangent to circle ω' at A intersects ω in D. Suppose that CD intersectsω and ω' in E and F, respectively (assume that E is between F and C). The perpendicular to AC from E intersects ω' in point P and perpendicular to AD from F intersectsω in point Q (The points A, P and Q lie on the same side of the line CD). Prove that the points A, P and Q are collinear.
Proposed by Mahdi Etesami Fard

2 In acute-angled triangle ABC, altitude of A meets BC at D, and M is midpoint of AC. Suppose that X is a point such that $\measuredangle AXB = \measuredangle DXM = 90^\circ$ (assume that X and C lie on opposite sides of the line BM). Show that $\measuredangle XMB = 2\measuredangle MBC$.Proposed by Davood Vakili

3 In a convex qualrilateral $ABCD$, let P be the intersection point of AD and BC. Suppose that I_1 and I_2 are the incenters of triangles PAB and PDC,respectively. Let O be the circumcenter of PAB, and H the orthocenter of PDC. Show that the circumcircles of triangles AI_1B and DHC are tangent together if and only if the circumcircles of triangles AOB and DI_2C are tangent

together.
Proposed by Hooman Fattahimoghaddam

---

4 In a convex quadrilateral $ABCD$, the lines AB and CD meet at point E and the lines AD and BC meet at point F. Let P be the intersection point of diagonals AC and BD. Suppose that ω_1 is a circle passing through D and tangent to AC at P. Also suppose that ω_2 is a circle passing through C and tangent to BD at P. Let X be the intersection point of ω_1 and AD, and Y be the intersection point of ω_2 and BC. Suppose that the circles ω_1 and ω_2 intersect each other in Q for the second time. Prove that the perpendicular from P to the line EF passes through the circumcenter of triangle XQY .
Proposed by Iman Maghsoudi

---

5 Do there exist six points $X_1, X_2, Y_1, Y_2, Z_1, Z_2$ in the plane such that all of the triangles $X_iY_jZ_k$ are similar for $1 \leq i, j, k \leq 2$?
Proposed by Morteza Saghafian
