Flanders Math Olympiad 1986

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1 A circle with radius R is divided into twelve equal parts. The twelve dividing points are connected with the centre of the circle, producing twelve rays. Starting from one of the dividing points a segment is drawn perpendicular to the next ray in the clockwise sense; from the foot of this perpendicular another perpendicular segment is drawn to the next ray, and the process is continued *ad infinitum*. What is the limit of the sum of these segments (in terms of R)?
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png

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2 Prove that for integer n we have:

$$n! \leq \left(\frac{n+1}{2}\right)^n$$

(please note that the pupils in the competition never heard of AM-GM or alikes, it is intended to be solved without any knowledge on inequalities)

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3 Let $\{a_k\}_{k\geq 0}$ be a sequence given by $a_0 = 0$, $a_{k+1} = 3 \cdot a_k + 1$ for $k \in \mathbb{N}$.

Prove that $11 \mid a_{155}$

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4 Given a cube in which you can put two massive spheres of radius 1.
What's the smallest possible value of the side - length of the cube?
Prove that your answer is the best possible.
