Flanders Math Olympiad 1987

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by parmenides51, Peter

1 A rectangle $ABCD$ is given. On the side AB, n different points are chosen strictly between A and B. Similarly, m different points are chosen on the side AD. Lines are drawn from the points parallel to the sides. How many rectangles are formed in this way? (One possibility is shown in the figure.)
https://cdn.artofproblemsolving.com/attachments/0/1/dcf48e4ce318fdcb8c7088a34fac226e26e24.png

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2 Two parallel lines a and b meet two other lines c and d. Let A and A' be the points of intersection of a with c and d, respectively. Let B and B' be the points of intersection of b with c and d, respectively. If X is the midpoint of the line segment AA' and Y is the midpoint of the segment BB', prove that

$$|XY| \leq \frac{|AB| + |A'B'|}{2}.$$

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3 Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x)^3 = -\frac{x}{12} \cdot \left(x^2 + 7x \cdot f(x) + 16 \cdot f(x)^2\right), \ \forall x \in \mathbb{R}.$$

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4 Show that for $p > 1$ we have

$$\lim_{n \to +\infty} \frac{1^p + 2^p + ... + (n-1)^p + n^p + (n-1)^p + ... + 2^p + 1^p}{n^2} = +\infty$$

Find the limit if $p = 1$.
