Flanders Math Olympiad 1989

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by Peter

1 Show that every subset of 1,2,...,99,100 with 55 elements contains at least 2 numbers with a difference of 9.

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2 When drawing all diagonals in a regular pentagon, one gets an smaller pentagon in the middle. What's the ratio of the areas of those pentagons?

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3 Show that:

$$\alpha = \pm \frac{\pi}{12} + k \cdot \frac{\pi}{2} (k \in \mathbb{Z}) \iff |\tan \alpha| + |\cot \alpha| = 4$$

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4 Let D be the set of positive reals different from 1 and let n be a positive integer.
If for $f : D \to \mathbb{R}$ we have $x^n f(x) = f(x^2)$,
and if $f(x) = x^n$ for $0 < x < \frac{1}{1989}$ and for $x > 1989$,
then prove that $f(x) = x^n$ for all $x \in D$.
