Flanders Math Olympiad 1990

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1 On the standard unit circle, draw 4 unit circles with centers $[0,1]$, $[1,0]$, $[0,-1]$, $[-1,0]$. You get a figure as below, find the area of the colored part.
`https://cdn.artofproblemsolving.com/attachments/6/1/a479365778b8a1d10373a47a4f698150e0633 png`

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2 Let a and b be two primes having at least two digits, such that $a>b$.

Show that

$$240|\left(a^4-b^4\right)$$

and show that 240 is the greatest positive integer having this property.

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3 We form a decimal code of 21 digits. the code may start with 0. Determine the probability that the fragment 0123456789 appears in the code.

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4 Let $f:\mathbb{R}_0^+\to\mathbb{R}_0^+$ be a strictly decreasing function.

(a) Be a_n a sequence of strictly positive reals so that $\forall k\in\mathbb{N}_0: k\cdot f(a_k)\geq (k+1)\cdot f(a_{k+1})$
Prove that a_n is ascending, that $\lim\limits_{k\to+\infty} f(a_k)$ = 0and that $\lim\limits_{k\to+\infty} a_k=+\infty$

(b) Prove that there exist such a sequence (a_n) in $\mathbb{R}_0^+$ if you know $\lim\limits_{x\to+\infty} f(x)=0$.
