Flanders Math Olympiad 1991

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1 Show that the number $111...111$ with 1991 times the number 1, is not prime.

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2 (a) Show that for every $n \in \mathbb{N}$ there is exactly one $x \in \mathbb{R}^+$ so that $x^n + x^{n+1} = 1$. Call this x_n.
(b) Find $\lim_{n \to +\infty} x_n$.

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3 Given ΔABC equilateral, with $X \in [A, B]$. Then we define unique points Y,Z so that $Y \in [B, C]$, $Z \in [A, C]$, ΔXYZ equilateral.

If $Area\left(\Delta ABC\right) = 2 \cdot Area\left(\Delta XYZ\right)$, find the ratio of $\frac{AX}{XB}, \frac{BY}{YC}, \frac{CZ}{ZA}$.

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4 A word of length n that consists only of the digits 0 and 1, is called a bit-string of length n. (For example, 000 and 01101 are bit-strings of length 3 and 5.) Consider the sequence $s(1), s(2), ...$ of bit-strings of length $n > 1$ which is obtained as follows :
(1) $s(1)$ is the bit-string $00...01$, consisting of $n-1$ zeros and a 1 ;
(2) $s(k+1)$ is obtained as follows :
(a) Remove the digit on the left of $s(k)$. This gives a bit-string t of length $n-1$.
(b) Examine whether the bit-string $t1$ (length n, adding a 1 after t) is already in $\{s(1), s(2), ..., s(k)\}$. If this is the not case, then $s(k+1) = t1$. If this is the case then $s(k+1) = t0$.

For example, if $n = 3$ we get : $s(1) = 001 \to s(2) = 011 \to s(3) = 111 \to s(4) = 110 \to s(5) = 101 \to s(6) = 010 \to s(7) = 100 \to s(8) = 000 \to s(9) = 001 \to ...$

Suppose $N = 2^n$.
Prove that the bit-strings $s(1), s(2), ..., s(N)$ of length n are all different.

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