Flanders Math Olympiad 1992

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1 For every positive integer n, determine the biggest positive integer k so that $2^k | \, 3^n + 1$

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2 It has come to a policeman's ears that 5 gangsters (all of different height) are meeting, one of them is the clan leader, he's the tallest of the 5. He knows the members will leave the building one by one, with a 10-minute break between them, and too bad for him Belgium has not enough policemen to follow all gangsters, so he's on his own to spot the clanleader, and he can only follow one member.

So he decides to let go the first 2 people, and then follow the first one that is taller than those two. What's the chance he actually catches the clan leader like this?

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3 A sphere is inscribed in a cone having apothema equal to A. The tangent circle of the sphere and the cone, determines the upper end of a cylinder which is inscribed in the sphere. Assume that the total area of the cone (mantle as well as base) equals nine times the area of a great circle of the sphere. Assume also that the apothema of the cone is larger than half the perimeter of the base of the cone. Determine the height of the cylinder as a function of A.
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4 Let A, B, P positive reals with $P \leq A + B$.
(a) Choose reals θ_1, θ_2 with $A\cos\theta_1 + B\cos\theta_2 = P$ and prove that

$$A\sin\theta_1 + B\sin\theta_2 \leq \sqrt{(A+B-P)(A+B+P)}$$

(b) Prove equality is attained when $\theta_1 = \theta_2 = \arccos\left(\frac{P}{A+B}\right)$.

(c) Take $A = \frac{1}{2}xy, B = \frac{1}{2}wz$ and $P = \frac{1}{4}\left(x^2+y^2-z^2-w^2\right)$ with $0 < x \leq y \leq x+z+w, z, w > 0$ and $z^2 + w^2 < x^2 + y^2$.
Show that we can translate (a) and (b) into the following theorem: from all quadrilaterals with (ordered) sidelenghts (x, y, z, w), the cyclical one has the greatest area.
