Flanders Math Olympiad 1993

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1 The 20 pupils in a class each send 10 cards to 10 (different) class members. *[note: you cannot send a card to yourself.]*
(a) Show at least 2 pupils sent each other a card.
(b) Now suppose we had n pupils sending m cards each. For which (m, n) is the above true?
(That is, find minimal $m(n)$ or maximal $n(m)$)

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2 A jeweler covers the diagonal of a unit square with small golden squares in the following way:
- the sides of all squares are parallel to the sides of the unit square
- for each neighbour is their sidelength either half or double of that square (squares are neighbour if they share a vertex)
- each midpoint of a square has distance to the vertex of the unit square equal to $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ of the diagonal. (so real length: $\times\sqrt{2}$)
- all midpoints are on the diagonal

(a) What is the side length of the middle square?
(b) What is the total gold-plated area?

`https://1.bp.blogspot.com/-azJkAVACPvQ/XWuytNaj27I/AAAAAAAAKpo/C6CONOzoQiYbFXfe41nNjNK8PF s400/1993%2Bflanders%2Bp2.png`

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3 For $a, b, c > 0$ we have:

$$-1 < \left(\frac{a-b}{a+b}\right)^{1993} + \left(\frac{b-c}{b+c}\right)^{1993} + \left(\frac{c-a}{c+a}\right)^{1993} < 1$$

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4 Define the sequence oa_n as follows: $oa_0 = 1, oa_n = oa_{n-1} \cdot cos\left(\frac{\pi}{2^{n+1}}\right)$.

Find $\lim\limits_{n\to+\infty} oa_n$.
