Flanders Math Olympiad 1996

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by Peter

1 In triangle ΔADC we got $AD = DC$ and $D = 100^\circ$.
In triangle ΔCAB we got $CA = AB$ and $A = 20^\circ$.

Prove that $AB = BC + CD$.

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2 Determine the gcd of all numbers of the form $p^8 - 1$, with p a prime above 5.

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3 Consider the points $1, \frac{1}{2}, \frac{1}{3}, ...$ on the real axis. Find the smallest value $k \in \mathbb{N}_0$ for which all points above can be covered with 5 **closed** intervals of length $\frac{1}{k}$.

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4 Consider a real poylnomial $p(x) = a_n x^n + ... + a_1 x + a_0$.
(a) If $\deg(p(x)) > 2$ prove that $\deg(p(x)) = 2 + deg(p(x+1) + p(x-1) - 2p(x))$.
(b) Let $p(x)$ a polynomial for which there are real constants r, s so that for all real x we have

$$p(x+1) + p(x-1) - rp(x) - s = 0$$

Prove $\deg(p(x)) \leq 2$.
(c) Show, in (b) that $s = 0$ implies $a_2 = 0$.
