Flanders Math Olympiad 1997

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1 Write the number 1997 as the sum of positive integers for which the product is maximal, and prove there's no better solution.

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2 In the cartesian plane, consider the curves $x^2 + y^2 = r^2$ and $(xy)^2 = 1$. Call F_r the convex polygon with vertices the points of intersection of these 2 curves. (if they exist)

(a) Find the area of the polygon as a function of r.
(b) For which values of r do we have a regular polygon?

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3 Δoa_1b_1 is isosceles with $\angle a_1ob_1 = 36^\circ$. Construct $a_2, b_2, a_3, b_3, ...$ as below, with $|oa_{i+1}| = |a_ib_i|$ and $\angle a_iob_i = 36^\circ$, Call the summed area of the first k triangles A_k. Let S be the area of the isocseles triangle, drawn in - - -, with top angle 108° and $|oc| = |od| = |oa_1|$, going through the points b_2 and a_2 as shown on the picture. (yes, cd is parallel to a_1b_1 there)
Show $A_k < S$ for every positive integer k.
`https://1.bp.blogspot.com/-Wi2fEsdckDE/XWuwIdw6hqI/AAAAAAAAKpc/_quN1EHOxYURpBfEgc8HiUN4bC`
`s400/1997%2Bflanders%2Bp3.png`

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4 Thirteen birds arrive and sit down in a plane. It's known that from each 5-tuple of birds, at least four birds sit on a circle. Determine the greatest $M \in \{1, 2, ..., 13\}$ such that from these 13 birds, at least M birds sit on a circle, but not necessarily $M + 1$ birds sit on a circle. (prove that your M is optimal)
