Spain Mathematical Olympiad 2004

www.artofproblemsolving.com/community/c459976

by OmicronGamma

–   Session 1

Problem 1   We have a set of 221 real numbers whose sum is 110721. It is deemed that the numbers form a rectangular table such that every row as well as the first and last columns are arithmetic progressions of more than one element. Prove that the sum of the elements in the four corners is equal to 2004.

Problem 2 $ABCD$ is a quadrilateral, P and Q are midpoints of the diagonals BD and AC, respectively. The lines parallel to the diagonals originating from P and Q intersect in the point O. If we join the four midpoints of the sides, X, Y, Z, and T, to O, we form four quadrilaterals: $OXBY$, $OYCZ$, $OZDT$, and $OTAX$. Prove that the four newly formed quadrilaterals have the same areas.

Problem 3   Represent for $\mathbb{Z}$ the set of all integers. Find all of the functions $f : \mathbb{Z} \to \mathbb{Z}$ such that for any x, y integers, they satisfy: $f(x + f(y)) = f(x) - y$.

–   Session 2

Problem 4   Does there exist such a power of 2, that when written in the decimal system its digits are all different than zero and it is possible to reorder the other digits to form another power of 2? Justify your answer.

Problem 5   Demonstrate that the condition necessary so that, in triangle ABC, the median from B is divided into three equal parts by the inscribed circumference of a circle is:
$A/5 = B/10 = C/13$.

Problem 6   We put, forming a circumference of a circle, 2004 bicolored files: white on one side of the file and black on the other. A movement consists in choosing a file with the black side upwards and flipping three files: the one chosen, the one to its right, and the one to its left. Suppose that initially there was only one file with its black side upwards. Is it possible, repeating the movement previously described, to get all of the files to have their white sides upwards? And if we were to have 2003 files, between which exactly one file began with the black side upwards?