Flanders Math Olympiad 2000

www.artofproblemsolving.com/community/c4601

by Peter, Arne

1   An integer consists of 7 different digits, and is a multiple of each of its digits.

What digits are in this nubmer?

2   Given two triangles and such that the lengths of the sides of the first triangle are the lengths of the medians of the second triangle. Determine the ratio of the areas of these triangles.

3   Let p_n be the n-th prime. $(p_1 = 2)$
Define the sequence (f_j) as follows:
- $f_1 = 1, f_2 = 2$
- $\forall j \geq 2$: if $f_j = kp_n$ for $k < p_n$ then $f_{j+1} = (k+1)p_n$
- $\forall j \geq 2$: if $f_j = p_n^2$ then $f_{j+1} = p_{n+1}$

(a) Show that all f_i are different
(b) from which index onwards are all f_i at least 3 digits?
(c) which integers do not appear in the sequence?
(d) how many numbers with less than 3 digits appear in the sequence?

4   Solve for $x \in [0, 2\pi[$:
$$\sin x < \cos x < \tan x < \cot x$$