Turkey EGMO TST 2017

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Day 1 9 February 2017

1 Let m, k, n be positive integers. Determine all triples (m, k, n) satisfying the following equation: $3^m 5^k = n^3 + 125$

2 At the beginning there are 2017 marbles in each of 1000 boxes. On each move Aybike chooses a box, grabs some of the marbles from that box and delivers them one for each to the boxes she wishes. At least how many moves does Aybike have to make to have different number of marbles in each box?

3 For all positive real numbers x, y, z satisfying the inequality

$$\frac{xy}{z} + \frac{yz}{x} + \frac{zx}{y} \leq 3,$$

prove that

$$\frac{x^2}{y^3} + \frac{y^2}{z^3} + \frac{z^2}{x^3} \geq \frac{x}{y} + \frac{y}{z} + \frac{z}{x}.$$

Day 2 10 February 2017

4 On the inside of the triangle ABC a point P is chosen with $\angle BAP = \angle CAP$. If $|AB| \cdot |CP| = |AC| \cdot |BP| = |BC| \cdot |AP|$, find all possible values of the angle $\angle ABP$.

5 In a 12×12 square table some stones are placed in the cells with at most one stone per cell. If the number of stones on each line, column, and diagonal is even, what is the maximum number of the stones?

Note. Each diagonal is parallel to one of two main diagonals of the table and consists of $1, 2 \ldots, 11$ or 12 cells.

6 Find all pairs of prime numbers (p, q), such that $\frac{(2p^2-1)^q+1}{p+q}$ and $\frac{(2q^2-1)^p+1}{p+q}$ are both integers.