Turkey Team Selection Test 2017

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Day 1 25 March 2017

1 m,n are positive integers and p is a prime number. Find all triples (m,n,p) satisfying $(m^3+n)(n^3+m)=p^3$

2 There are two-way flights between some of the 2017 cities in a country, such that given two cities, it is possible to reach one from the other. No matter how the flights are appointed, one can define k cities as "special city", so that there is a direct flight from each city to at least one "special city". Find the minimum value of k.

3 At the ABC triangle the midpoints of BC, AC, AB are respectively D, E, F and the triangle tangent to the incircle at G, H and I in the same order.The midpoint of AD is J. BJ and AG intersect at point K. The $C-$centered circle passing through A cuts the $[CB$ ray at point X. The line passing through K and parallel to the BC and AX meet at U. IU and BC intersect at the P point. There is Y point chosen at incircle. PY is tangent to incircle at point Y. Prove that D, E, F, Y are cyclic.

Day 2 26 March 2017

4 Each two of n students, who attended an activity, have different ages. It is given that each student shook hands with at least one student, who did not shake hands with anyone younger than the other. Find all possible values of n.

5 For all positive real numbers a,b,c with $a+b+c=3$, show that

$$a^3b+b^3c+c^3a+9\geq 4(ab+bc+ca).$$

6 Prove that no pair of different positive integers (m,n) exist, such that $\frac{4m^2n^2-1}{(m^2-n^2)^2}$ is an integer.

Day 3 27 March 2017

7 Let a be a real number. Find the number of functions $f:\mathbb{R}\rightarrow\mathbb{R}$ depending on a, such that $f(xy+f(y))=f(x)y+a$ holds for every $x,y\in\mathbb{R}$.

8 In a triangle ABC the bisectors through vertices B and C meet the sides $[AC]$ and $[AB]$ at D and E respectively. Let I_c be the center of the excircle which is tangent to the side $[AB]$ and F the midpoint of $[BI_c]$. If $|CF|^2 = |CE|^2 + |DF|^2$, show that ABC is an equilateral triangle.

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9 Let S be a set of finite number of points in the plane any 3 of which are not linear and any 4 of which are not concyclic. A coloring of all the points in S to red and white is called *discrete coloring* if there exists a circle which encloses all red points and excludes all white points. Determine the number of *discrete colorings* for each set S.
