Italy TST 1996

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1	1-Let A and B be two diametrically opposite points on a circle with radius 1. Points $P_1, P_2, ..., P_n$ are arbitrarily chosen on the circle. Let a and b be the geometric means of the distances of $P_1, P_2, ..., P_n$ from A and B, respectively. Show that at least one of the numbers a and b does not exceed $\sqrt{2}$
2	2. Let $A_1, A_2, ..., A_n$be distinct subsets of an n-element set X ($n \geq 2$). Show that there exists an element x of X such that the sets $A_1 \setminus \{x\}$,:....., $A_n \setminus \{x\}$ are all distinct.
3	3.Let ABCD be a parallelogram with side AB longer than AD and acute angle $\angle DAB$. The bisector of DAB meets side CD at L and line BC at K. If O is the circumcenter of triangle LCK, prove that the points B,C,O,D lie on a circle.
4	4.4. Prove that there exists a set X of 1996 positive integers with the following properties: (i) the elements of X are pairwise coprime; (ii) all elements of X and all sums of two or more distinct elements of X are composite numbers