Spain Mathematical Olympiad 2003

www.artofproblemsolving.com/community/c460828

by OmicronGamma

– Session 1

Problem 1 Prove that for any prime p, different than 2 and 5, there exists such a multiple of p whose digits are all nines. For example, if $p = 13$, such a multiple is $999999 = 13 * 76923$.

Problem 2 Does there exist such a finite set of real numbers M that has at least two distinct elements and has the property that for two numbers, a, b, belonging to M, the number $2a - b^2$ is also an element in M?

Problem 3 The altitudes of the triangle ABC meet in the point H. You know that $AB = CH$. Determine the value of the angle $\widehat{BCA}$.

– Session 2

Problem 4 Let x be a real number such that $x^3 + 2x^2 + 10x = 20$. Demonstrate that both x and x^2 are irrational.

Problem 5 How many possible areas are there in a convex hexagon with all of its angles being equal and its sides having lengths $1, 2, 3, 4, 5$ and 6, in any order?

Problem 6 We string $2n$ white balls and $2n$ black balls, forming a continuous chain. Demonstrate that, in whatever order the balls are placed, it is always possible to cut a segment of the chain to contain exactly n white balls and n black balls.