

South africa National Olympiad 1996

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- 1 Find the highest power of 2 that divides exactly into $1996! = 1 \times 2 \times \cdots \times 1996$.

- 2 Find all real numbers for which $3^x + 4^x = 5^x$.

- 3 The sides of triangle ABC has integer lengths. Given that $AC = 6$ and $\angle BAC = 120^\circ$, determine the lengths of the other two sides.

- 4 In the Rainbow Nation there are two airways: Red Rockets and Blue Boeings. For any two cities in the Rainbow Nation it is possible to travel from the one to the other using either or both of the airways. It is known, however, that it is impossible to travel from Beanville to Mieliestad using only Red Rockets - not directly nor by travelling via other cities. Show that, using only Blue Boeings, one can travel from any city to any other city by stopping at at most one city along the way.

- 5 ABC is a triangle with sides 1, 2 and $\sqrt{3}$. Determine the smallest possible area of an equilateral triangle with a vertex on each side of triangle ABC .

- 6 The function f is increasing and convex (i.e. every straight line between two points on the graph of f lies above the graph) and satisfies $f(f(x)) = 3^x$ for all $x \in \mathbb{R}$. If $f(0) = 0.5$ determine $f(0.75)$ with an error of at most 0.025. The following are corrent to the number of digits given:

$$3^{0.25} = 1.31607, \quad 3^{0.50} = 1.73205, \quad 3^{0.75} = 2.27951.$$