

South africa National Olympiad 1998

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1 Is $\log_{10} 8$ rational?

2 Find the maximum value of

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma$$

where α, β and γ are positive and $\alpha + \beta + \gamma = 180^\circ$.

3 A, B, C, D, E and F lie (in that order) on the circumference of a circle. The chords AD, BE and CF are concurrent. P, Q and R are the midpoints of AD, BE and CF respectively. Two further chords $AG \parallel BE$ and $AH \parallel CF$ are drawn. Show that PQR is similar to DGH .

4 In a group of people, every two people have exactly one friend in common. Prove that there is a person who is a friend of everyone else.

5 Prove that

$$\gcd \left(\binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n-1} \right)$$

is a prime if n is a power of a prime, and 1 otherwise.

6 You are given n squares, not necessarily all of the same size, which have total area 1. Is it always possible to place them without overlapping in a square of area 2?
