Spain Mathematical Olympiad 2002

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– Session 1

Problem 1 Find all the polynomials $P(t)$ of one variable that fullfill the following for all real numbers x and y: $P(x^2-y^2)=P(x+y)P(x-y)$.

Problem 2 In the triangle ABC, A' is the foot of the altitude to A, and H is the orthocenter. $a)$ Given a positive real number $k=\frac{AA'}{HA'}$, find the relationship between the angles B and C, as a function of k. $b)$ If B and C are fixed, find the locus of the vertice A for any value of k.

Problem 3 The function g is defined about the natural numbers and satisfies the following conditions: $g(2)=1$ $g(2n)=g(n)$ $g(2n+1)=g(2n)+1$.
Where n is a natural number such that $1\leq n\leq 2002$.

Find the maximum value M of $g(n)$. Also, calculate how many values of n satisfy the condition of $g(n)=M$.

– Session 2

Problem 4 Denote n as a natural number, and m as the result of writing the digits of n in reverse order. Determine, if they exist, the numbers of three digits which satisfy $2m+S=n$, S being the sum of the digits of n.

Problem 5 Consider 2002 segments on a plane, such that their lengths are the same. Prove that there exists such a straight line r such that the sum of the lengths of the projections of the 2002 segments about r is less than $\frac{2}{3}$.

Problem 6 In a regular polygon H of $6n+1$ sides (n is a positive integer), we paint r vertices red, and the rest blue. Demonstrate that the number of isosceles triangles that have three of their vertices of the same color does not depend on the way we distribute the colors on the vertices of H.