South africa National Olympiad 2006

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by djb86

1 Reduce the fraction

$$\frac{2121212121210}{1121212121211}$$

to its simplest form.

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2 Triangle ABC has $BC = 1$ and $AC = 2$. What is the maximum possible value of $\hat{A}$.

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3 Determine all positive integers whose squares end in 196.

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4 In triangle ABC, $AB = AC$ and $B\hat{A}C = 100^\circ$. Let D be on AC such that $A\hat{B}D = C\hat{B}D$. Prove that $AD + DB = BC$.

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5 Find the number of subsets X of $\{1, 2, \dots, 10\}$ such that X contains at least two elements and such that no two elements of X differ by 1.

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6 Consider the function f defined by

$$f(n) = \frac{1}{n}\left(\left\lfloor\frac{n}{1}\right\rfloor + \left\lfloor\frac{n}{2}\right\rfloor + \dots + \left\lfloor\frac{n}{n}\right\rfloor\right)$$

for all positive integers n. (Here $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.) Prove that

(a) $f(n+1) > f(n)$ for infinitely many n.

(b) $f(n+1) < f(n)$ for infinitely many n.
