South africa National Olympiad 2013

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by DylanN

1 2013 is the first year since the Middle Ages that consists of four consecutive digits. How many such years are there still to come after 2013 (and before 10000)?

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2 A is a two-digit number and B is a three-digit number such that A increased by B

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3 Let ABC be an acute-angled triangle and AD one of its altitudes (D on BC). The line through D parallel to AB is denoted by l, and t is the tangent to the circumcircle of ABC at A. Finally, let E be the intersection of l and t. Show that CE and t are perpendicular to each other.

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4 Determine all pairs of polynomials f and g with real coefficients such that

$$x^2 \cdot g(x) = f(g(x)).$$

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5 Some coins are placed on a 20×13 board. Two coins are called *neighbours* if they are in the same row or column and no other coins lie between them. What is the largest number of coins that can be placed on the board if no coin is allowed to have more than two neighbours?

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6 Let ABC be an acute-angled triangle with $AC \neq BC$, and let O be the circumcentre and F the foot of the altitude through C. Furthermore, let X and Y be the feet of the perpendiculars dropped from A and B respectively to (the extension of) CO. The line FO intersects the circumcircle of FXY a second time at P. Prove that $OP < OF$.
