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–	Grade 10
–	**Day 1**
1	Suppose that the sequence $\{a_n\}$ satisfy $a_1 = 1$ and $a_{2k} = a_{2k-1} + a_k, \quad a_{2k+1} = a_{2k}$ for $k = 1, 2, \ldots$ Prove that $a_{2^n} < 2^{\frac{n^2}{2}}$ for any integer $n \geq 3$.
2	Let I be the incenter of $\triangle ABC$ with $AB > AC$. Let Γ be the circle with diameter AI. The circumcircle of $\triangle ABC$ intersects Γ at points A, D, with point D lying on $\overset{\frown}{AC}$ (not containing B). Let the line passing through A and parallel to BC intersect Γ at points A, E. If DI is the angle bisector of $\angle CDE$, and $\angle ABC = 33^\circ$, find the value of $\angle BAC$.
3	Can you make 2015 positive integers $1, 2, \ldots, 2015$ to be a certain permutation which can be ordered in the circle such that the sum of any two adjacent numbers is a multiple of 4 or a multiple of 7?
4	For any positive integer n, we have the set $P_n = \{n^k \mid k = 0, 1, 2, \ldots\}$. For positive integers a, b, c, we define the group of (a, b, c) as lucky if there is a positive integer m such that $a - 1$, $ab - 12$, $abc - 2015$ (the three numbers need not be different from each other) belong to the set P_m. Find the number of lucky groups.
–	**Day 2**
5	Suppose that a, b are real numbers, function $f(x) = ax + b$ satisfies $\mid f(x) \mid \leq 1$ for any $x \in [0, 1]$. Find the range of values of $S = (a+1)(b+1)$.
6	In $\triangle ABC$, we have three edges with lengths $BC = a$, $CA = b$ $AB = c$, and $c < b < a < 2c$. P and Q are two points of the edges of $\triangle ABC$, and the straight line PQ divides $\triangle ABC$ into two parts with the same area. Find the minimum value of the length of the line segment PQ.
7	In $\triangle ABC$, we have $AB > AC > BC$. D, E, F are the tangent points of the inscribed circle of $\triangle ABC$ with the line segments AB, BC, AC respectively. The points L, M, N are the midpoints of the line segments DE, EF, FD. The straight line NL intersects with ray AB at P, straight line LM intersects ray BC at Q and the straight line NM intersects ray AC at R. Prove that $PA \cdot QB \cdot RC = PD \cdot QE \cdot RF$.

8 For any integers m, n, we have the set $A(m,n) = \{x^2 + mx + n \mid x \in \mathbb{Z}\}$, where $\mathbb{Z}$ is the integer set. Does there exist three distinct elements a, b, c which belong to $A(m,n)$ and satisfy the equality $a = bc$?

– Grade 11

– Day 1

1 Grade 10 P2

2 Given a sequence $\{a_n\}_{n\in\mathbb{Z}^+}$ defined by $a_1 = 1$ and $a_{2k} = a_{2k-1} + a_k, a_{2k+1} = a_{2k}$ for all positive integer k.

Prove that, for any positive integer n, $a_{2^n} > 2^{\frac{n^2}{4}}$.

3 Grade 10 P4

4 Given 8 pairwise distinct positive integers $a_1, a_2, , a_8$ such that the greatest common divisor of any three of them is equal to 1. Show that there exists positive integer $n \geq 8$ and n pairwise distinct positive integers $m_1, m_2, , m_n$ with the greatest common divisor of all n numbers equal to 1 such that for any positive integers $1 \leq p < q < r \leq n$, there exists positive integers $1 \leq i < j \leq 8$ that $a_i a_j \mid m_p + m_q + m_r$.

– Day 2

5 Given two points E and F lie on segment AB and AD, respectively. Let the segments BF and DE intersects at point C. If its known that $AE + EC = AF + FC$, show that $AB + BC = AD + DC$.

6 Given a positive integer $n \geq 2$. Let $A = \{(a,b) \mid a, b \in \{1, 2, , n\}\}$ be the set of points in Cartesian coordinate plane. How many ways to colour points in A, each by one of three fixed colour, such that, for any $a, b \in \{1, 2, , n-1\}$, if (a,b) and $(a+1,b)$ have same colour, then $(a, b+1)$ and $(a+1, b+1)$ also have same colour.

7 Grade 10 P7

8 Find all prime number p such that there exists an integer-coefficient polynomial $f(x) = x^{p-1} + a_{p-2}x^{p-2} + \ + a_1 x + a_0$ that has $p-1$ consecutive positive integer roots and $p^2 \mid f(i)f(-i)$, where i is the imaginary unit.