

## **AoPS Community**

## 2007 Belarusian National Olympiad

www.artofproblemsolving.com/community/c467451 by RagvaloD

<ul> <li>Find all polynomials with degree ≤ n and nonnegative coefficients, such that P(x)P(1/x) P(1)<sup>2</sup> for every positive x</li> <li>Circles S1 and S2 with centers O1 and O2, respectively, pass through the centers of each oth Let A be one of their intersection points. Two points M1 and M2 begin to move simultaneous starting from A. Point M1 moves along S1 and point M2 moves along S2. Both points move clockwise direction and have the same linear velocity v.</li> <li>(a) Prove that all triangles AM1M2 are equilateral.</li> <li>(b) Determine the trajectory of the movement of the center of the triangle AM1M2 and find i linear velocity.</li> <li>Given a 2n × 2m table (m, n ∈ N) with one of two signs + or - in each of its cells. A unit of all the cells of some row and some column is called a cross. The cell on the intersection this row and this column is called the center of the cross. The following procedure we call transformation of the table: we mark all cells which contain and then, in turn, we replace to signs in all cells of the crosses which centers are marked by the opposite signs. (It is easy see that the order of the choice of the crosses doesnt matter.) We call a table attainable if can be obtained from some table applying such transformations one time. Find the number of all attainable tables.</li> <li>Each point of a circle is painted in one of the N colors (N ≥ 2). Prove that there exists a inscribed trapezoid such that all its vertices are painted the same color.</li> <li>Day 2</li> <li>Let O be the intersection point of the diagonals of the convex quadrilateral ABCD, AO = C Points P and Q are marked on the segments AO and CO, respectively, such that PO = OQ. L N and K be the intersection points of the sides AB, CD, and the lines DP and BQ respective Prove that the points N, O, and K are colinear.</li> <li>Let a be the sum and b the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -\frac{11}{10} and a &gt; <sup>6</sup>/<sub>11</sub>.</li> </ul>	-	Day 1
<ul> <li>2 Circles S₁ and S₂ with centers O₁ and O₂, respectively, pass through the centers of each oth Let A be one of their intersection points. Two points M₁ and M₂ begin to move simultaneous starting from A. Point M₁ moves along S₁ and point M₂ moves along S₂. Both points move clockwise direction and have the same linear velocity v.</li> <li>(a) Prove that all triangles AM₁M₂ are equilateral.</li> <li>(b) Determine the trajectory of the movement of the center of the triangle AM₁M₂ and find linear velocity.</li> <li>3 Given a 2n × 2m table (m, n ∈ N) with one of two signs + or - in each of its cells. A unit of all the cells of some row and some column is called a cross. The cell on the intersectin this row and this column is called the center of the cross. The following procedure we call transformation of the table: we mark all cells which contain and then, in turn, we replace th signs in all cells of the crosses which centers are marked by the opposite signs. (It is easy see that the order of the choice of the crosses doesnt matter.) We call a table attainable if can be obtained from some table applying such transformations one time. Find the number of all attainable tables.</li> <li>4 Each point of a circle is painted in one of the N colors (N ≥ 2). Prove that there exists a inscribed trapezoid such that all its vertices are painted the same color.</li> <li>- Day 2</li> <li>5 Let O be the intersection point of the diagonals of the convex quadrilateral ABCD, AO = C Points P and Q are marked on the segments AO and CO, respectively, such that PO = OQ. L N and K be the intersection points of the real coust AO and CO, respectively, such that PO = OQ. L N and K be the intersection points of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that the points N, O, and K are colinear.</li> <li>6 Let a be the sum and b the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -110 and a &gt; 610.</li> </ul>	1	Find all polynomials with degree $\leq n$ and nonnegative coefficients, such that $P(x)P(\frac{1}{x}) \leq P(1)^2$ for every positive $x$
<ul> <li>(a) Prove that all triangles AM1M2 are equilateral.</li> <li>(b) Determine the trajectory of the movement of the center of the triangle AM1M2 and find i linear velocity.</li> <li>3 Given a 2n × 2m table (m, n ∈ N) with one of two signs + or - in each of its cells. A unit of all the cells of some row and some column is called a cross. The cell on the intersectin this row and this column is called the center of the cross. The following procedure we call transformation of the table: we mark all cells which contain and then, in turn, we replace the signs in all cells of the crosses which centers are marked by the opposite signs. (It is easy see that the order of the choice of the crosses doesnt matter.) We call a table attainable if can be obtained from some table applying such transformations one time. Find the number of all attainable tables.</li> <li>4 Each point of a circle is painted in one of the N colors (N ≥ 2). Prove that there exists a inscribed trapezoid such that all its vertices are painted the same color.</li> <li>- Day 2</li> <li>5 Let O be the intersection point of the diagonals of the convex quadrilateral ABCD, AO = C Points P and Q are marked on the segments AO and CO, respectively, such that PO = OQ. L N and K be the intersection points of the sides AB, CD, and the lines DP and BQ respective Prove that the points N, O, and K are colinear.</li> <li>6 Let a be the sum and b the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -110 and a &gt; 6/11.</li> </ul>	2	Circles $S_1$ and $S_2$ with centers $O_1$ and $O_2$ , respectively, pass through the centers of each other. Let $A$ be one of their intersection points. Two points $M_1$ and $M_2$ begin to move simultaneously starting from $A$ . Point $M_1$ moves along $S_1$ and point $M_2$ moves along $S_2$ . Both points move in clockwise direction and have the same linear velocity $v$ .
<ul> <li>Given a 2n × 2m table (m, n ∈ N) with one of two signs + or - in each of its cells. A unio of all the cells of some row and some column is called a cross. The cell on the intersectin this row and this column is called the center of the cross. The following procedure we call transformation of the table: we mark all cells which contain and then, in turn, we replace the signs in all cells of the crosses which centers are marked by the opposite signs. (It is easy see that the order of the choice of the crosses doesnt matter.) We call a table attainable if can be obtained from some table applying such transformations one time. Find the number of all attainable tables.</li> <li>Each point of a circle is painted in one of the N colors (N ≥ 2). Prove that there exists a inscribed trapezoid such that all its vertices are painted the same color.</li> <li>Day 2</li> <li>Let O be the intersection point of the diagonals of the convex quadrilateral ABCD, AO = C. Points P and Q are marked on the segments AO and CO, respectively, such that PO = OQ. L N and K be the intersection points of the sides AB, CD, and the lines DP and BQ respective Prove that the points N, O, and K are colinear.</li> <li>Let a be the sum and b the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -11/10 and a &gt; 6/11.</li> </ul>		(a) Prove that all triangles $AM_1M_2$ are equilateral. (b) Determine the trajectory of the movement of the center of the triangle $AM_1M_2$ and find its linear velocity.
<ul> <li>4 Each point of a circle is painted in one of the <i>N</i> colors (<i>N</i> ≥ 2). Prove that there exists a inscribed trapezoid such that all its vertices are painted the same color.</li> <li>- Day 2</li> <li>5 Let <i>O</i> be the intersection point of the diagonals of the convex quadrilateral <i>ABCD</i>, <i>AO</i> = <i>C</i>. Points <i>P</i> and <i>Q</i> are marked on the segments <i>AO</i> and <i>CO</i>, respectively, such that <i>PO</i> = <i>OQ</i>. Let <i>N</i> and <i>K</i> be the intersection points of the sides <i>AB</i>, <i>CD</i>, and the lines <i>DP</i> and <i>BQ</i> respective Prove that the points <i>N</i>, <i>O</i>, and <i>K</i> are collinear.</li> <li>6 Let <i>a</i> be the sum and <i>b</i> the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -<sup>11</sup>/<sub>10</sub> and a &gt; <sup>6</sup>/<sub>11</sub>.</li> </ul>	3	Given a $2n \times 2m$ table $(m, n \in \mathbb{N})$ with one of two signs + or - in each of its cells. A union of all the cells of some row and some column is called a cross. The cell on the intersectin of this row and this column is called the center of the cross. The following procedure we call a transformation of the table: we mark all cells which contain and then, in turn, we replace the signs in all cells of the crosses which centers are marked by the opposite signs. (It is easy to see that the order of the choice of the crosses doesnt matter.) We call a table attainable if it can be obtained from some table applying such transformations one time. Find the number of all attainable tables.
<ul> <li>Day 2</li> <li>Let <i>O</i> be the intersection point of the diagonals of the convex quadrilateral <i>ABCD</i>, <i>AO</i> = <i>C</i>. Points <i>P</i> and <i>Q</i> are marked on the segments <i>AO</i> and <i>CO</i>, respectively, such that <i>PO</i> = <i>OQ</i>. Let <i>N</i> and <i>K</i> be the intersection points of the sides <i>AB</i>, <i>CD</i>, and the lines <i>DP</i> and <i>BQ</i> respective Prove that the points <i>N</i>, <i>O</i>, and <i>K</i> are colinear.</li> <li>Let <i>a</i> be the sum and <i>b</i> the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -<sup>11</sup>/<sub>10</sub> and a &gt; <sup>6</sup>/<sub>11</sub>.</li> </ul>	4	Each point of a circle is painted in one of the $N$ colors ( $N \ge 2$ ). Prove that there exists an inscribed trapezoid such that all its vertices are painted the same color.
<ul> <li>5 Let <i>O</i> be the intersection point of the diagonals of the convex quadrilateral <i>ABCD</i>, <i>AO</i> = <i>C</i> Points <i>P</i> and <i>Q</i> are marked on the segments <i>AO</i> and <i>CO</i>, respectively, such that <i>PO</i> = <i>OQ</i>. Let <i>N</i> and <i>K</i> be the intersection points of the sides <i>AB</i>, <i>CD</i>, and the lines <i>DP</i> and <i>BQ</i> respective Prove that the points <i>N</i>, <i>O</i>, and <i>K</i> are colinear.</li> <li>6 Let <i>a</i> be the sum and <i>b</i> the product of the real roots of the equation x<sup>4</sup> - x<sup>3</sup> - 1 = 0 Prove that b &lt; -<sup>11</sup>/<sub>10</sub> and a &gt; <sup>6</sup>/<sub>11</sub>.</li> </ul>	-	Day 2
<b>6</b> Let <i>a</i> be the sum and <i>b</i> the product of the real roots of the equation $x^4 - x^3 - 1 = 0$ Prove that $b < -\frac{11}{10}$ and $a > \frac{6}{11}$ .	5	Let <i>O</i> be the intersection point of the diagonals of the convex quadrilateral <i>ABCD</i> , $AO = CO$ . Points <i>P</i> and <i>Q</i> are marked on the segments <i>AO</i> and <i>CO</i> , respectively, such that $PO = OQ$ . Let <i>N</i> and <i>K</i> be the intersection points of the sides <i>AB</i> , <i>CD</i> , and the lines <i>DP</i> and <i>BQ</i> respectively. Prove that the points <i>N</i> , <i>O</i> , and <i>K</i> are colinear.
	6	Let <i>a</i> be the sum and <i>b</i> the product of the real roots of the equation $x^4 - x^3 - 1 = 0$ Prove that $b < -\frac{11}{10}$ and $a > \frac{6}{11}$ .

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**7** Find solution in positive integers :

$$n^5 + n^4 = 7^m - 1$$

8 Let (m, n) be a pair of positive integers.
(a) Prove that the set of all positive integers can be partitioned into four pairwise disjoint nonempty subsets such that none of them has two numbers with absolute value of their difference equal to either m, n, or m + n.
(b) Find all pairs (m, n) such that the set of all positive integers can not be partitioned into three pairwise disjoint nonempty subsets satisfying the above condition.

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