Finals 1995

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	Day 1
1	How many subsets of $\{1, 2, ..., 2n\}$ do not contain two numbers with sum $2n + 1$?
2	The diagonals of a convex pentagon divide it into a small pentagon and ten triangles. What is the largest number of the triangles that can have the same area?
3	Let p be a prime number, and define a sequence by: $x_i = i$ for $i =, 0, 1, 2..., p - 1$ and $x_n = x_{n-1} + x_{n-p}$ for $n \geq p$ Find the remainder when x_{p^3} is divided by p.
	Day 2
1	The positive reals $x_1, x_2, ..., x_n$ have harmonic mean 1. Find the smallest possible value of $x_1 + \frac{x_2^2}{2} + \frac{x_3^3}{3} + ... + \frac{x_n^n}{n}$.
2	An urn contains n balls labeled $1, 2, ..., n$. We draw the balls out one by one (without replacing them) until we obtain a ball whose number is divisible by k. Find all k such that the expected number of balls removed is k.
3	PA, PB, PC are three rays in space. Show that there is just one pair of points B', C' with B' on the ray PB and C' on the ray PC such that $PC' + B'C' = PA + AB'$ and $PB' + B'C' = PA + AC'$.