Finals 2000

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Day 1

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1 Find number of solutions in non-negative reals to the following equations:

$$x_1 + x_n^2 = 4x_n$$
$$x_2 + x_1^2 = 4x_1$$
$$...$$
$$x_n + x_{n-1}^2 = 4x_{n-1}$$

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2 Let a triangle ABC satisfy $AC = BC$; in other words, let ABC be an isosceles triangle with base AB. Let P be a point inside the triangle ABC such that $\angle PAB = \angle PBC$. Denote by M the midpoint of the segment AB. Show that $\angle APM + \angle BPC = 180^\circ$.

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3 The sequence $p_1, p_2, p_3, ...$ is defined as follows. p_1 and p_2 are primes. p_n is the greatest prime divisor of $p_{n-1} + p_{n-2} + 2000$. Show that the sequence is bounded.

Day 2

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1 $PA_1A_2...A_n$ is a pyramid. The base $A_1A_2...A_n$ is a regular n-gon. The apex P is placed so that the lines PA_i all make an angle $60^\cdot$ with the plane of the base. For which n is it possible to find B_i on PA_i for $i = 2, 3, ..., n$ such that $A_1B_2 + B_2B_3 + B_3B_4 + ... + B_{n-1}B_n + B_nA_1 < 2A_1P$?

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2 In the unit squre For the given natural number $n \geq 2$ find the smallest number k that from each set of k unit squares of the nxn chessboard one can achoose a subset such that the number of the unit squares contained in this subset an lying in a row or column of the chessboard is even

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3 Show that the only polynomial of odd degree satisfying $p(x^2 - 1) = p(x)^2 - 1$ for all x is $p(x) = x$
