**Finals 2013**

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**Day 1**

**1** Find all solutions of the following equation in integers $x, y : x^4 + y = x^3 + y^2$

**2** There are given integers $a$ and $b$ such that $a$ is different from $0$ and the number $3 + a + b^2$ is divisible by $6a$. Prove that $a$ is negative.

**3** Given is a quadrilateral $ABCD$ in which we can inscribe circle. The segments $AB, BC, CD$ and $DA$ are the diameters of the circles $o1, o2, o3$ and $o4$, respectively. Prove that there exists a circle tangent to all of the circles $o1, o2, o3$ and $o4$.

**Day 2**

**4** Given is a tetrahedron $ABCD$ in which $AB = CD$ and the sum of measures of the angles $BAD$ and $BCD$ equals $180$ degrees. Prove that the measure of the angle $BAD$ is larger than the measure of the angle $ADC$.

**5** Let k,m and n be three different positive integers. Prove that

$$\left(k - \frac{1}{k}\right)\left(m - \frac{1}{m}\right)\left(n - \frac{1}{n}\right) \le kmn - (k + m + n).$$

**6** For each positive integer $n$ determine the maximum number of points in space creating the set $A$ which has the following properties: $1)$ the coordinates of every point from the set $A$ are integers from the range $[0, n]$ $2)$ for each pair of different points $(x_1, x_2, x_3), (y_1, y_2, y_3)$ belonging to the set $A$ it is satisfied at least one of the following inequalities $x_1 < y_1, x_2 < y_2, x_3 < y_3$ and at least one of the following inequalities $x_1 > y_1, x_2 > y_2, x_3 > y_3$.