Vietnam National Olympiad 1985

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by April

Day 1

1 Find all pairs (x, y) of integers such that $x^3 - y^3 = 2xy + 8$.

2 Find all functions $f : \mathbb{Z} \mapsto \mathbb{R}$ which satisfy:
i) $f(x)f(y) = f(x + y) + f(x - y)$ for all integers x, y
ii) $f(0) \neq 0$
iii) $f(1) = \frac{5}{2}$

3 A parallelepiped with the side lengths a, b, c is cut by a plane through its intersection of diagonals which is perpendicular to one of these diagonals. Calculate the area of the intersection of the plane and the parallelepiped.

Day 2

1 Let a, b and m be positive integers. Prove that there exists a positive integer n such that $(a^n - 1)b$ is divisible by m if and only if $\gcd(ab, m) = \gcd(b, m)$.

2 Find all real values of parameter a for which the equation in x

$$16x^4 - ax^3 + (2a + 17)x^2 - ax + 16 = 0$$

has four solutions which form an arithmetic progression.

3 A triangular pyramid $O.ABC$ with base ABC has the property that the lengths of the altitudes from A, B and C are not less than $\frac{OB+OC}{2}$, $\frac{OC+OA}{2}$ and $\frac{OA+OB}{2}$, respectively. Given that the area of ABC is S, calculate the volume of the pyramid.