Vietnam National Olympiad 1997

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Day 1

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1 Given a circle (O,R). A point P lies inside the circle, OP=d, d¡R. We consider quadrilaterals ABCD, inscribed in (O), such that AC is perp to BD at point P. Evaluate the maximum and minimum values of the perimeter of ABCD in terms of R and d.

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2 Let n be an integer which is greater than 1, not divisible by 1997.
Let $a_m = m + \frac{mn}{1997}$ for all m=1,2,..,1996 $b_m = m + \frac{1997m}{n}$ for all m=1,2,..,n-1
We arrange the terms of two sequence $(a_i), (b_j)$ in the ascending order to form a new sequence $c_1 \le c_2 \le ... \le c_{1995+n}$
Prove that $c_{k+1} - c_k < 2$ for all k=1,2,...,1994+n

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3 Find the number of functions $f : \mathbb{N} \to \mathbb{N}$ which satisfying:
(i) $f(1) = 1$
(ii) $f(n)f(n+2) = f^2(n+1) + 1997$ for every natural numbers n.

Day 2

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1 Let $k = \sqrt[3]{3}$.
a, Find all polynomials $p(x)$ with rationl coefficients whose degree are as least as possible such that $p(k + k^2) = 3 + k$.
b, Does there exist a polynomial $p(x)$ with integer coefficients satisfying $p(k + k^2) = 3 + k$

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2 Prove that for evey positive integer n, there exits a positive integer k such that $2^n | 19^k - 97$

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3 In the unit cube, given 75 points, no three of which are collinear. Prove that there exits a triangle whose vertices are among the given points and whose area is not greater than 7/72.
