Junior Balkan MO 2017

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by sqing

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1 Determine all the sets of six consecutive positive integers such that the product of some two of them . added to the product of some other two of them is equal to the product of the remaining two numbers.

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2 Let x, y, z be positive integers such that $x \neq y \neq z \neq x$ .Prove that

$$(x+y+z)(xy+yz+zx-2) \geq 9xyz.$$

When does the equality hold?

Proposed by Dorlir Ahmeti, Albania

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3 Let ABC be an acute triangle such that $AB \neq AC$,with circumcircle Γ and circumcenter O. Let M be the midpoint of BC and D be a point on Γ such that $AD \perp BC$. let T be a point such that $BDCT$ is a parallelogram and Q a point on the same side of BC as A such that $\angle BQM = \angle BCA$ and $\angle CQM = \angle CBA$. Let the line AO intersect Γ at E $(E \neq A)$ and let the circumcircle of $\triangle ETQ$ intersect Γ at point $X \neq E$. Prove that the point A, M and X are collinear.

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4 Consider a regular 2n-gon P,$A_1, A_2, \cdots, A_{2n}$ in the plane ,where n is a positive integer . We say that a point S on one of the sides of P can be seen from a point E that is external to P , if the line segment SE contains no other points that lie on the sides of P except S .We color the sides of P in 3 different colors (ignore the vertices of P,we consider them colorless), such that every side is colored in exactly one color, and each color is used at least once . Moreover ,from every point in the plane external to P , points of most 2 different colors on P can be seen .Find the number of distinct such colorings of P (two colorings are considered distinct if at least one of sides is colored differently).

Proposed by Viktor Simjanoski, Macedonia

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