Vietnam National Olympiad 2007

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by hien

Day 1	
1	Solve the system of equations: $\begin{cases} (1+\frac{12}{3x+y}).\sqrt{x}=2 \\ (1-\frac{12}{3x+y}).\sqrt{y}=6 \end{cases}$
2	Let x,y be integer number with $x,y \neq -1$ so that $\frac{x^4-1}{y+1}+\frac{y^4-1}{x+1} \in \mathbb{Z}$. Prove that $x^4y^{44}-1$ is divisble by $x+1$
3	Let B,C be fixed points and A be roving point. Let H, G be orthecentre and centroid of triagle ABC. Known midpoint of HG lies on BC, find locus of A
Day 2	
1	Given a regular 2007-gon. Find the minimal number k such that: Among every k vertexes of the polygon, there always exists 4 vertexes forming a convex quadrilateral such that 3 sides of the quadrilateral are also sides of the polygon.
2	Given a number $b>0$, find all functions $f:\mathbb{R}\to\mathbb{R}$ such that: $f(x+y)=f(x).3^{b^y+f(y)-1}+b^x.\left(3^{b^y+f(y)-1}-b^y\right)\forall x,y\in\mathbb{R}$
3	Let ABCD be trapezium that is inscribed in circle (O) with larger edge BC. P is a point lying outer segment BC. PA cut (O) at N(that means PA isn't tangent of (O)), the circle with diameter PD intersect (O) at E, DE meet BC at N. Prove that MN always pass through a fixed point.