Vietnam National Olympiad 2008

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1 Determine the number of solutions of the simultaneous equations $x^2 + y^3 = 29$ and $\log_3 x \cdot \log_2 y = 1.$

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2 Given a triangle with acute angle $\angle BEC,$ let E be the midpoint of $AB.$ Point M is chosen on the opposite ray of EC such that $\angle BME = \angle ECA.$ Denote by θ the measure of angle $\angle BEC.$ Evaluate $\frac{MC}{AB}$ in terms of $\theta.$

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3 Let $m = 2007^{2008},$ how many natural numbers n are there such that $n < m$ and $n(2n+1)(5n+2)$ is divisible by m (which means that $m \mid n(2n + 1)(5n + 2)$) ?

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4 he sequence of real number (x_n) is defined by $x_1 = 0, x_2 = 2$ and $x_{n+2} = 2^{-x_n} + \frac{1}{2}\ \forall n = 1, 2, 3 \ldots$ Prove that the sequence has a limit as n approaches $+\infty.$ Determine the limit.

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5 What is the total number of natural numbes divisible by 9 the number of digits of which does not exceed 2008 and at least two of the digits are 9s?

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6 Let x, y, z be distinct non-negative real numbers. Prove that

$$\frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} \geq \frac{4}{xy + yz + zx}.$$

When does the equality hold?

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7 Let AD is centroid of ABC triangle. Let (d) is the perpendicular line with AD. Let M is a point on (d). Let E, F are midpoints of MB, MC respectively. The line through point E and perpendicular with (d) meet AB at P. The line through point F and perpendicular with (d) meet AC at Q. Let (d') is a line through point M and perpendicular with PQ. Prove (d') always pass a fixed point.
