Vietnam National Olympiad 2009

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1 **Problem 1.**Find all (x,y) such that:

$$\{\begin{array}{ccc}\frac{1}{\sqrt{1+2x^2}}+\frac{1}{\sqrt{1+2y^2}} & = & \frac{2}{\sqrt{1+2xy}}\\ \sqrt{x(1-2x)}+\sqrt{y(1-2y)} & = & \frac{2}{9}\end{array}$$

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2 The sequence $\{x_n\}$ is defined by

$$\begin{cases} x_1=\frac{1}{2} \\ x_n=\frac{\sqrt{x_{n-1}^2+4x_{n-1}}+x_{n-1}}{2}\end{cases}$$

Prove that the sequence $\{y_n\}$, where $y_n=\sum_{i=1}^{n}\frac{1}{{x_i}^2}$, has a finite limit and find that limit.

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3 Let A, B be two fixed points and C is a variable point on the plane such that $\angle ACB=\alpha$ (constant) ($0^{\circ}\le\alpha\le 180^{\circ}$). Let D, E, F be the projections of the incenter I of triangle ABC to its sides BC, CA, AB, respectively. Denoted by M, N the intersections of AI, BI with EF, respectively. Prove that the length of the segment MN is constant and the circumcircle of triangle DMN always passes through a fixed point.

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4 Let a, b, c be three real numbers. For each positive integer number n, $a^n+b^n+c^n$ is an integer number. Prove that there exist three integers p, q, r such that a, b, c are the roots of the equation $x^3+px^2+qx+r=0$.

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5 Let $S=\{1,2,3,\ldots,2n\}$ ($n\in\mathbb{Z}^+$). Ddetermine the number of subsets T of S such that there are no 2 element in T a,b such that $|a-b|=\{1,n\}$
