



Vietnam National Olympiad 2010

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1 Solve the system equations

$$\begin{cases} x^4 - y^4 = 240 \\ x^3 - 2y^3 = 3(x^2 - 4y^2) - 4(x - 8y) \end{cases}$$

2 Let $\{a_n\}$ be a sequence which satisfy

$$a_1 = 5 \text{ and } a_n = \sqrt[n]{a_{n-1}^{n-1} + 2^{n-1} + 2 \cdot 3^{n-1}} \quad \forall n \geq 2$$

(a) Find the general formula for a_n

(b) Prove that $\{a_n\}$ is decreasing sequences

3 In plane, let a circle (O) and two fixed points B, C lies in (O) such that BC not is the diameter. Consider a point A varies in (O) such that $A \neq B, C$ and $AB \neq AC$. Call D and E respective is intersect of BC and internal and external bisector of \widehat{BAC} , I is midpoint of DE . The line that pass through orthocenter of $\triangle ABC$

and perpendicular with AI intersects AD, AE respective at M, N .

1/Prove that MN pass through a fixed point

2/Determine the place of A such that S_{AMN} has maximum value

4 Prove that for each positive integer n , the equation

$$x^2 + 15y^2 = 4^n$$

has at least n integer solution (x, y)

5 Let a positive integer n . Consider square table 3×3 . One use n colors to color all cells of table such that each cell is colored by exactly one color.

Two colored table is same if we can receive them from other by a rotation through center of 3×3 table

How many ways to color this square table satisfies above conditions.

