

Vietnam Team Selection Test 1985

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Day 1

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- 1 The sequence (x_n) of real numbers is defined by $x_1 = \frac{29}{10}$ and $x_{n+1} = \frac{x_n}{\sqrt{x_n^2 - 1}} + \sqrt{3}$ for all $n \geq 1$. Find a real number a (if exists) such that $x_{2k-1} > a > x_{2k}$.
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- 2 Let ABC be a triangle with $AB = AC$. A ray Ax is constructed in space such that the three planar angles of the trihedral angle $ABCx$ at its vertex A are equal. If a point S moves on Ax , find the locus of the incenter of triangle SBC .
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- 3 Does there exist a triangle ABC satisfying the following two conditions:
 (a) $\sin^2 A + \sin^2 B + \sin^2 C = \cot A + \cot B + \cot C$
 (b) $S \geq a^2 - (b - c)^2$ where S is the area of the triangle ABC .
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Day 2

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- 1 A convex polygon A_1, A_2, \dots, A_n is inscribed in a circle with center O and radius R so that O lies inside the polygon. Let the inradii of the triangles $A_1A_2A_3, A_1A_3A_4, \dots, A_1A_{n-1}A_n$ be denoted by r_1, r_2, \dots, r_{n-2} . Prove that $r_1 + r_2 + \dots + r_{n-2} \leq R(n \cos \frac{\pi}{n} - n + 2)$.
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- 2 Find all real values of a for which the equation $(a - 3x^2 + \cos \frac{9\pi x}{2})\sqrt{3 - ax} = 0$ has an odd number of solutions in the interval $[-1, 5]$
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- 3 Suppose a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(f(x)) = -x$ for all $x \in \mathbb{R}$. Prove that f has infinitely many points of discontinuity.
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