Vietnam Team Selection Test 1996

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Day 1

1 In the plane we are given $3 \cdot n$ points ($n >$1) no three collinear, and the distance between any two of them is ≤ 1. Prove that we can construct n pairwise disjoint triangles such that: The vertex set of these triangles are exactly the given 3n points and the sum of the area of these triangles $< 1/2$.

2 For each positive integer n, let $f(n)$ be the maximal natural number such that: $2^{f(n)}$ divides $\sum_{i=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \binom{n}{2 \cdot i+1} 3^i$. Find all n such that $f(n) = 1996$.

For each positive integer n, let $f(n)$ be the maximal natural number such that: $2^{f(n)}$ divides $\sum_{i=1}^{n+1/2} \binom{2 \cdot i+1}{n}$. Find all n such that $f(n) = 1996$.

3 Find the minimum value of the expression:

$$f(a,b,c) = (a+b)^4 + (b+c)^4 + (c+a)^4 - \frac{4}{7} \cdot (a^4+b^4+c^4).$$

Day 2

1 Given 3 non-collinear points A, B, C. For each point M in the plane (ABC) let M_1 be the point symmetric to M with respect to AB, M_2 be the point symmetric to M_1 with respect to BC and M' be the point symmetric to M_2 with respect to AC. Find all points M such that MM' obtains its minimum. Let this minimum value be d. Prove that d does not depend on the order of the axes of symmetry we chose (we have 3 available axes, that is BC, CA, AB. In the first part the order of axes we chose AB, BC, CA, and the second part of the problem states that the value d doesn't depend on this order).

2 There are some people in a meeting; each doesn't know at least 56 others, and for any pair, there exist a third one who knows both of them. Can the number of people be 65?

3 Find all reals a such that the sequence $\{x(n)\}$, $n = 0, 1, 2, \ldots$ that satisfy: $x(0) = 1996$ and $x_{n+1} = \frac{a}{1+x(n)^2}$ for any natural number n has a limit as n goes to infinity.