Vietnam Team Selection Test 2001

www.artofproblemsolving.com/community/c4756

by orl, darij grinberg, grobber, al.M.V., hxtung

Day 1

---

1 Let a sequence of integers $\{a_n\}$, $n \in \mathbb{N}$ be given, defined by

$$a_0 = 1, a_n = a_{n-1} + a_{[n/3]}$$

for all $n \in \mathbb{N}^*$.

Show that for all primes $p \leq 13$, there are infinitely many integer numbers k such that a_k is divided by p.
(Here $[x]$ denotes the integral part of real number x).

---

2 In the plane let two circles be given which intersect at two points A, B; Let PT be one of the two common tangent line of these circles (P, T are points of tangency). Tangents at P and T of the circumcircle of triangle APT meet each other at S. Let H be a point symmetric to B under PT. Show that A, S, H are collinear.

---

3 Some club has 42 members. Its known that among 31 arbitrary club members, we can find one pair of a boy and a girl that they know each other. Show that from club members we can choose 12 pairs of knowing each other boys and girls.

Day 2

---

1 Lets consider the real numbers a, b, c satisfying the condition

$$21 \cdot a \cdot b + 2 \cdot b \cdot c + 8 \cdot c \cdot a \leq 12.$$

Find the minimal value of the expression

$$P(a, b, c) = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

---

2 Let an integer $n > 1$ be given. In the space with orthogonal coordinate system $Oxyz$ we denote by T the set of all points (x, y, z) with x, y, z are integers, satisfying the condition: $1 \leq x, y, z \leq n$. We paint all the points of T in such a way that: if the point $A(x_0, y_0, z_0)$ is painted then points

$B(x_1, y_1, z_1)$ for which $x_1 \leq x_0, y_1 \leq y_0$ and $z_1 \leq z_0$ could not be painted. Find the maximal number of points that we can paint in such a way the above mentioned condition is satisfied.

---

3 Let a sequence $\{a_n\}$, $n \in \mathbb{N}^*$ given, satisfying the condition

$$0 < a_{n+1} - a_n \leq 2001$$

for all $n \in \mathbb{N}^*$

Show that there are infinitely many pairs of positive integers (p, q) such that $p < q$ and a_p is divisor of a_q.
