**ELMO Problems 2017**

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by cjquines0, whatshisbucket

**Day 1** June 10th

**1** Let $a_1, a_2, \ldots, a_n$ be positive integers with product $P$, where $n$ is an odd positive integer. Prove that

$$\gcd(a_1^n + P, a_2^n + P, \ldots, a_n^n + P) \le 2\gcd(a_1, \ldots, a_n)^n.$$

*Proposed by Daniel Liu*

**2** Let $ABC$ be a triangle with orthocenter $H$, and let $M$ be the midpoint of $\overline{BC}$. Suppose that $P$ and $Q$ are distinct points on the circle with diameter $\overline{AH}$, different from $A$, such that $M$ lies on line $PQ$. Prove that the orthocenter of $\triangle APQ$ lies on the circumcircle of $\triangle ABC$.

*Proposed by Michael Ren*

**3** nic$\kappa$y is drawing kappas in the cells of a square grid. However, he does not want to draw kappas in three consecutive cells (horizontally, vertically, or diagonally). Find all real numbers $d > 0$ such that for every positive integer $n$, nic$\kappa$y can label at least $dn^2$ cells of an $n \times n$ square.

*Proposed by Mihir Singhal and Michael Kural*

**Day 2** June 17th

**4** An integer $n > 2$ is called *tasty* if for every ordered pair of positive integers $(a, b)$ with $a + b = n$, at least one of $\frac{a}{b}$ and $\frac{b}{a}$ is a terminating decimal. Do there exist infinitely many tasty integers?

*Proposed by Vincent Huang*

**5** The edges of $K_{2017}$ are each labeled with $1, 2,$ or $3$ such that any triangle has sum of labels at least $5$. Determine the minimum possible average of all $\binom{2017}{2}$ labels.

(Here $K_{2017}$ is defined as the complete graph on 2017 vertices, with an edge between every pair of vertices.)

*Proposed by Michael Ma*

 

**6** Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real numbers $a, b,$ and $c$:

(i) If $a + b + c \geq 0$ then $f(a^3) + f(b^3) + f(c^3) \geq 3f(abc)$.

(ii) If $a + b + c \leq 0$ then $f(a^3) + f(b^3) + f(c^3) \leq 3f(abc)$.

*Proposed by Ashwin Sah*

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