Vietnam Team Selection Test 2005

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Day 1

1 Let $(I),(O)$ be the incircle, and, respectiely, circumcircle of ABC. (I) touches BC,CA,AB in D,E,F respectively. We are also given three circles $\omega_a,\omega_b,\omega_c$, tangent to $(I),(O)$ in D,K (for ω_a), E,M (for ω_b), and F,N (for ω_c).

a) Show that DK,EM,FN are concurrent in a point P;

b) Show that the orthocenter of DEF lies on OP.

2 Given n chairs around a circle which are marked with numbers from 1 to n .There are k, $k\leq 4\cdot n$ students sitting on those chairs .Two students are called neighbours if there is no student sitting between them. Between two neighbours students ,there are at less 3 chairs. Find the number of choices of k chairs so that k students can sit on those and the condition is satisfied.

3 Find all functions $f:\mathbb{Z}\mapsto\mathbb{Z}$ satisfying the condition: $f(x^3+y^3+z^3)=f(x)^3+f(y)^3+f(z)^3$.

Day 2

1 Let be given positive reals a,b,c. Prove that: $\frac{a^3}{(a+b)^3}+\frac{b^3}{(b+c)^3}+\frac{c^3}{(c+a)^3}\geq\frac{3}{8}$.

2 Let $p\in\mathbb{P},p>3$. Calcute:

a)$S=\sum_{k=1}^{\frac{p-1}{2}}\left[\frac{2k^2}{p}\right]-2\cdot\left[\frac{k^2}{p}\right]$ if $p\equiv 1 \mod 4$

b) $T=\sum_{k=1}^{\frac{p-1}{2}}\left[\frac{k^2}{p}\right]$ if $p\equiv 1 \mod 8$

3 n is called *diamond 2005* if $n=\overline{...ab999...99999cd...}$, e.g. 2005×9. Let $\{a_n\}:a_n<C\cdot n$, $\{a_n\}$ is increasing. Prove that $\{a_n\}$ contain infinite *diamond 2005*.

Compare with this problem. (`http://www.mathlinks.ro/Forum/topic-15091.html`)