Vietnam Team Selection Test 2006

www.artofproblemsolving.com/community/c4761

by nmtruong1986, duongqua

Day 1

---

1 Given an acute angles triangle ABC, and H is its orthocentre. The external bisector of the angle $\angle BHC$ meets the sides AB and AC at the points D and E respectively. The internal bisector of the angle $\angle BAC$ meets the circumcircle of the triangle ADE again at the point K. Prove that HK is through the midpoint of the side BC.

---

2 Find all pair of integer numbers (n, k) such that n is not negative and k is greater than 1, and satisfying that the number:

$$A = 17^{2006n} + 4.17^{2n} + 7.19^{5n}$$

can be represented as the product of k consecutive positive integers.

---

3 In the space are given 2006 distinct points, such that no 4 of them are coplanar. One draws a segment between each pair of points.

A natural number m is called *good* if one can put on each of these segments a positive integer not larger than m, so that every triangle whose three vertices are among the given points has the property that two of this triangle's sides have equal numbers put on, while the third has a larger number put on.

Find the minimum value of a *good* number m.

Day 2

---

1 Prove that for all real numbers $x, y, z \in [1, 2]$ the following inequality always holds:

$$(x + y + z)(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}) \geq 6(\frac{x}{y + z} + \frac{y}{z + x} + \frac{z}{x + y}).$$

When does the equality occur?

---

2 Given a non-isoceles triangle ABC inscribes a circle (O, R) (center O, radius R). Consider a varying line l such that $l \perp OA$ and l always intersects the rays AB, AC and these intersectional points are called M, N. Suppose that the lines BN and CM intersect, and if the intersectional point is called K then the lines AK and BC intersect. 1, Assume that P is the intersectional point of AK and BC. Show that the circumcircle of the triangle MNP is always through a fixed point. 2, Assume that H is the orthocentre of the triangle AMN. Denote $BC = a$, and d is the distance between A and the line HK. Prove that $d \leq \sqrt{4R^2 - a^2}$ and the equality occurs iff the line l is through the intersectional point of two lines AO and BC.

---

3 The real sequence $\{a_n|n=0,1,2,3,...\}$ defined $a_0=1$ and

$$a_{n+1}=\frac{1}{2}\left(a_n+\frac{1}{3\cdot a_n}\right).$$

Denote

$$A_n=\frac{3}{3\cdot a_n^2-1}.$$

Prove that A_n is a perfect square and it has at least n distinct prime divisors.
