Vietnam Team Selection Test 2008

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Day 1

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1 On the plane, given an angle xOy. M be a mobile point on ray Ox and N a mobile point on ray Oy. Let d be the external angle bisector of angle xOy and I be the intersection of d with the perpendicular bisector of MN. Let P, Q be two points lie on d such that $IP = IQ = IM = IN$, and let K the intersection of MQ and NP.
1. Prove that K always lie on a fixed line.
2. Let d_1 line perpendicular to IM at M and d_2 line perpendicular to IN at N. Assume that there exist the intersections E, F of d_1, d_2 from d. Prove that EN, FM and OK are concurrent.

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2 Find all values of the positive integer m such that there exists polynomials $P(x), Q(x), R(x, y)$ with real coefficient satisfying the condition: For every real numbers a, b which satisfying $a^m - b^2 = 0$, we always have that $P(R(a, b)) = a$ and $Q(R(a, b)) = b$.

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3 Let an integer $n > 3$. Denote the set $T = \{1, 2, \ldots, n\}$. A subset S of T is called *wanting set* if S has the property: There exists a positive integer c which is not greater than $\frac{n}{2}$ such that $|s_1 - s_2| \neq c$ for every pairs of arbitrary elements $s_1, s_2 \in S$. How many does a *wanting set* have at most are there ?

Day 2

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1 Let m and n be positive integers. Prove that $6m|(2m + 3)^n + 1$ if and only if $4m|3^n + 1$

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2 Let k be a positive real number. Triangle ABC is acute and not isosceles, O is its circumcenter and AD,BE,CF are the internal bisectors. On the rays AD,BE,CF, respectively, let points L,M,N such that $\frac{AL}{AD} = \frac{BM}{BE} = \frac{CN}{CF} = k$. Denote $(O_1), (O_2), (O_3)$ be respectively the circle through L and touches OA at A, the circle through M and touches OB at B, the circle through N and touches OC at C.
1) Prove that when $k = \frac{1}{2}$, three circles $(O_1), (O_2), (O_3)$ have exactly two common points, the centroid G of triangle ABC lies on that common chord of these circles.
2) Find all values of k such that three circles $(O_1), (O_2), (O_3)$ have exactly two common points

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3 Consider the set $M = \{1, 2, \ldots, 2008\}$. Paint every number in the set M with one of the three colors blue, yellow, red such that each color is utilized to paint at least one number. Define two sets:

$S_1 = \{(x,y,z) \in M^3 \mid x, y, z$ have the same color and $2008|(x+y+z)\}$; $S_2 = \{(x,y,z) \in M^3 \mid x, y, z$ have three pairwisely different colors and $2008|(x+y+z)\}$.

Prove that $2|S_1| > |S_2|$ (where $|X|$ denotes the number of elements in a set X).