Vietnam Team Selection Test 2009

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Day 1

1 Let an acute triangle ABC with curcumcircle (O). Call A_1, B_1, C_1 are foots of perpendicular line from A, B, C to opposite side. A_2, B_2, C_2 are reflect points of A_1, B_1, C_1 over midpoints of BC, CA, AB respectively. Circle $(AB_2C_2), (BC_2A_2), (CA_2B_2)$ cut (O) at A_3, B_3, C_3 respectively. Prove that: A_1A_3, B_1B_3, C_1C_3 are concurent.

2 Let a polynomial $P(x) = rx^3 + qx^2 + px + 1$ $(r > 0)$ such that the equation $P(x) = 0$ has only one real root. A sequence (a_n) is defined by $a_0 = 1, a_1 = -p, a_2 = p^2 - q, a_{n+3} = -pa_{n+2} - qa_{n+1} - ra_n$.
Prove that (a_n) contains an infinite number of nagetive real numbers.

3 Let a, b be positive integers. a, b and a.b are not perfect squares.

Prove that at most one of following equations

$ax^2 - by^2 = 1$ and $ax^2 - by^2 = -1$

has solutions in positive integers.

Day 2

1 Let a, b, c be positive numbers.Find k such that: $(k + \frac{a}{b+c})(k + \frac{b}{c+a})(k + \frac{c}{a+b}) \geq (k + \frac{1}{2})^3$

2 Let a circle (O) with diameter AB. A point M move inside (O). Internal bisector of $\widehat{AMB}$ cut (O) at N, external bisector of $\widehat{AMB}$ cut NA, NB at P, Q. AM, BM cut circle with diameter NQ, NP at R, S.
Prove that: median from N of triangle NRS pass over a fix point.

3 There are $6n+4$ mathematicians participating in a conference which includes $2n+1$ meetings. Each meeting has one round table that suits for 4 people and n round tables that each table suits for 6 people. We have known that two arbitrary people sit next to or have opposite places doesn't exceed one time.
1. Determine whether or not there is the case $n = 1$.
2. Determine whether or not there is the case $n > 1$.