2013 International Physics Online Olympiad

www.artofproblemsolving.com/community/c47644

by jgf1123, ahaanomegas

– The Inaugural Contest

---

1 A construction rope is tied to two trees. It is straight and taut. It is then vibrated at a constant velocity v_1. The tension in the rope is then halved. Again, the rope is vibrated at a constant velocity v_2. The tension in the rope is then halved again. And, for the third time, the rope is vibrated at a constant velocity, this time v_3. The value of $\frac{v_1}{v_3}+\frac{v_3}{v_1}$ can be expressed as a positive number $\frac{m\sqrt{r}}{n}$, where m and n are relatively prime, and r is not divisible by the square of any prime. Find $m+n+r$. If the number is rational, let $r=1$.

(Ahaan Rungta, 2 points)

---

2 One hundred billion light years from Earth is planet Glorp. The inhabitants of Glorp are intelligent, uniform, amorphous beings with constant density which can modify their shape in any way, and reproduce by splitting. Suppose a Glorpian has somehow formed itself into a spinning cylinder in a frictionless environment. It then splits itself into two Glorpians of equal mass, which proceed to mold themselves into cylinders of the same height, but not the same radius, as the original Glorpian. If the new Glorpians' angular velocities after this are equal and the angular velocity of the original Glorpian is ω, let the angular velocity of the each of the new Glorpians be ω'. Then, find $\left(\frac{\omega'}{\omega}\right)^{10}$.

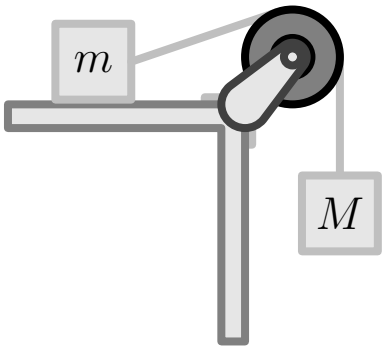
(B. Dejean, 3 points)

---

3 A rigid (solid) cylinder is put at the top of a frictionless 25°-to-the-horizontal incline that is 3.0 m high. It is then released so that it rolls down the incline. If v is the speed at the bottom of the incline, what is v^2, in m^2/s^2?

(B. Dejean and Ahaan Rungta, 3 points)
Note: Since there is no friction, the cylinder cannot roll, and thus the problem is flawed. Two answers were accepted and given full credit.

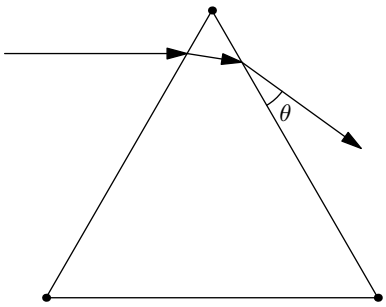
4



A pulley system of two blocks, shown above, is released from rest. The block on the table, which has mass $m = 1.0$ kg slides after the time of release and hits the pulley to come to a dead stop. There was originally a distance of 1.0 m between the block and the pulley, which the block fully covers during the slide. From the time of release to the time of hitting the pulley, the angle that the rope above the table makes with the horizontal axis is a, nearly constant, 10.0°. The hanging block has mass $M = 2.0$ kg. The table has a coefficient of friction of 0.50 with the block that sits on it. The pulley is frictionless. Also, assume that, during the entire slide, the block never leaves the ground. Let t be the number of seconds in takes for the 1.0-m slide. Find $100t$, rounded to two significant figures.

(Ahaan Rungta, 4 points)

5



The cross-section of a prism with index of refraction 1.5 is an equilateral triangle, as shown above. A ray of light comes in horizontally from air into the prism, and has the opportunity to leave the prism, at an angle θ with respect to the surface of the triangle. Find θ in degrees and round to the nearest whole number.

(Ahaan Rungta, 5 points)

---

6 A particle with charge $8.0\,\mu$C and mass 17 g enters a magnetic field of magnitude 7.8 mT perpendicular to its non-zero velocity. After 30 seconds, let the absolute value of the angle between its initial velocity and its current velocity, in radians, be θ. Find 100θ.

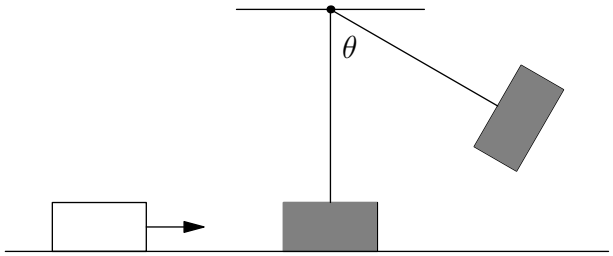
(B. Dejean, 5 points)

---

7 Ancient astronaut theorist Nutter B. Butter claims that the Caloprians from planet Calop, 30 light years away and at rest with respect to the Earth, wiped out the dinosaurs. The iridium layer in the crust, he claims, indicates spaceships with the fuel necessary to travel at 30

(B. Dejean, 6 points)

8



A block of mass $m = 4.2$ kg slides through a frictionless table with speed v and collides with a block of identical mass m, initially at rest, that hangs on a pendulum as shown above. The collision is perfectly elastic and the pendulum block swings up to an angle $\theta = 12°$, as labeled in the diagram. It takes a time $t = 1.0$ s for the block to swing up to this peak.Find $10v$, in m/s and round to the nearest integer. Do not approximate $\theta \approx 0$; however, assume θ is small enough as to use the small-angle approximation for the period of the pendulum.

(Ahaan Rungta, 6 points)

---

9 Bob, a spherical person, is floating around peacefully when Dave the giant orange fish launches him straight up 23 m/s with his tail. If Bob has density 100 kg/m^3, let $f(r)$ denote how far underwater his centre of mass plunges underwater once he lands, assuming his centre of mass was at water level when he's launched up. Find $\lim_{r\to 0}(f(r))$. Express your answer is meters and round to the nearest integer. Assume the density of water is 1000 kg/m^3.

(B. Dejean, 6 points)

---

10 Two masses are connected with spring constant k. The masses have magnitudes m and M. The center-of-mass of the system is fixed. If $k = 100$ N/m and $m = \frac{1}{2}M = 1$ kg, let the ground state energy of the system be E. If E can be expressed in the form $a \times 10^p$ eV (electron-volts),

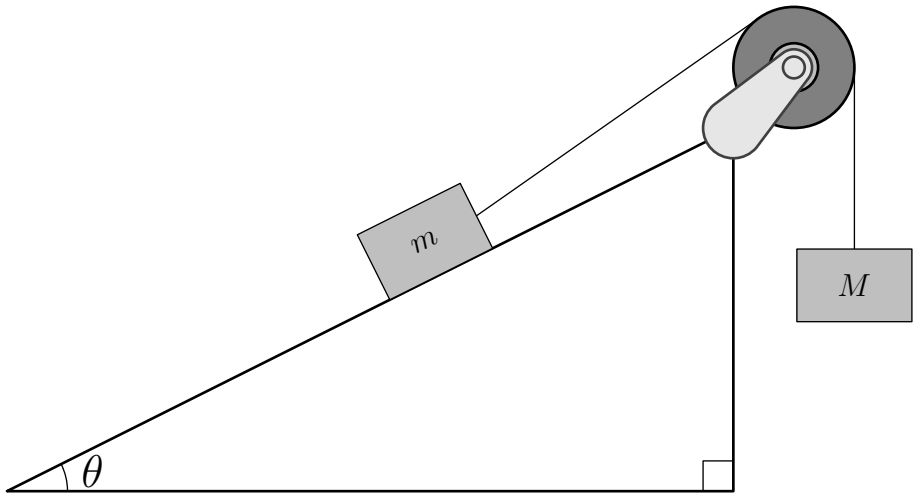
find the ordered pair (a, p), where $0 < a < 10$, and it is rounded to the nearest positive integer and p is an integer. For example, 4.2×10^7 should be expressed as $(4, 7)$.

(Trung Phan, 10 points)

– WOOT! Contest

---

1 A block of mass m on a frictionless inclined plane of angle θ is connected by a cord over a small frictionless, massless pulley to a second block of mass M hanging vertically, as shown. If $M = 1.5m$, and the acceleration of the system is $\frac{g}{3}$, where g is the acceleration of gravity, what is θ, in degrees, rounded to the nearest integer?



(Proposed by Ahaan Rungta)

---

2 Light of a blue laser (wavelength $\lambda = 475$ nm) goes through a narrow slit which has width d. After the light emerges from the slit, it is visible on a screen that is 2.013 m away from the slit. The distance between the center of the screen and the first minimum band is 765 mm. Find the width of the slit d, in nanometers.

(Proposed by Ahaan Rungta)

---

3 Let the rest energy of a particle be E. Let the work done to increase the speed of this particle from rest to v be W. If $W = \frac{13}{40}E$, then $v = kc$, where k is a constant. Find $10000k$ and round to the nearest whole number.

(Proposed by Ahaan Rungta)

---

4 The Iphoon particle, of charge q, is accelerated from rest by a potential difference of V. This

strange particle then enters a region with a uniform magnetic field, B, which is perpendicular to the particle's velocity. The Iphoon follows a circular path with radius R. If $q = 1\,\mu$C, $V = 1$ kV, $B = 1$ mT, and $R = 2$ ft, let the weight of an Iphoon, in Newtons, be w. If $w \approx 10^p$, where p is an integer, find p. That is, what is the order of magnitude of the weight?

(Proposed by Ahaan Rungta)

---

5 A uniform ladder of mass m and length $\mathcal{L}$ is resting on a wall. A man of mass m climbs up the ladder and is in perfect equilibrium with the ladder when he is $\frac{2}{3}\mathcal{L}$ the way up the ladder. The ladder makes an angle of $\theta = 30^\circ$ with the horizontal floor. If the coefficient of static friction between the ladder and the wall is the same as that between the ladder and the floor, which is μ, what is μ, expressed to the nearest thousandth?

(Proposed by Ahaan Rungta)

---

6 A fancy bathroom scale is calibrated in Newtons. This scale is put on a ramp, which is at a 40° angle to the horizontal. A box is then put on the scale and the box-scale system is then pushed up the ramp by a horizontal force F. The system slides up the ramp at a constant speed. If the bathroom scale reads R and the coefficient of static friction between the system and the ramp is 0.40, what is $\frac{F}{R}$? Round to the nearest thousandth.

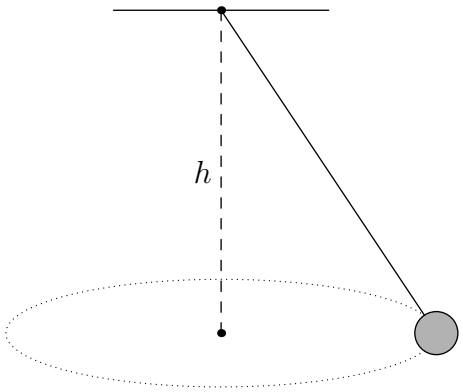
(Proposed by Ahaan Rungta)

---

7 A conical pendulum is formed from a rope of length 0.50 m and negligible mass, which is suspended from a fixed pivot attached to the ceiling. A ping-pong ball of mass 3.0 g is attached to the lower end of the rope. The ball moves in a circle with constant speed in the horizontal plane and the ball goes through one revolution in 1.0 s. How high is the ceiling in comparison to the horizontal plane in which the ball revolves? Express your answer to two significant digits, in cm.

(Proposed by Ahaan Rungta)

During the WOOT Contest, contestants wondered what exactly a conical pendulum looked like. Since contestants were not permitted to look up information during the contest, we posted this diagram:



The question is to find h.

---

8 A right-triangulated prism made of benzene sits on a table. The hypotenuse makes an angle of 30° with the horizontal table. An incoming ray of light hits the hypotenuse horizontally, and leaves the prism from the vertical leg at an acute angle of γ with respect to the vertical leg. Find γ, in degrees, to the nearest integer. The index of refraction of benzene is 1.50.

(Proposed by Ahaan Rungta)

---

9 A massless string is wrapped around a frictionless pulley of mass M. The string is pulled down with a force of 50 N, so that the pulley rotates due to the pull. Consider a point P on the rim of the pulley, which is a solid cylinder. The point has a constant linear (tangential) acceleration component equal to the acceleration of gravity on Earth, which is where this experiment is being held. What is the weight of the cylindrical pulley, in Newtons?

(Proposed by Ahaan Rungta)

This problem was not fully correct. Within friction, the pulley cannot rotate. So we responded:

> Excellent observation! This is very true. To submit, I'd say just submit as if it were rotating and ignore friction. In some effects such as these, I'm pretty sure it turns out that friction doesn't change the answer much anyway, but, yes, just submit as if it were rotating and you are just ignoring friction.

So do this problem imagining that the pulley does rotate somehow.

---

10 A young baseball player thinks he has hit a home run and gets excited, but, instead, he has just hit it to an outfielder who is just able to catch the ball, and does so at ground level. The ball was hit at a height of 1.5 meters from the ground at an angle ϕ above the horizontal axis. The catch

was taken at a horizontal distance 30 meters from home plate, which was where the batter hit the ball. The ball left the bat at a speed of 21 m/s. Find all possible values $0 < \phi < 90^\circ$, in degrees, rounded to the nearest integer. You may use WolframAlpha, Mathematica, or a graphing aid to compute ϕ after you derive an expression to solve for it.

(Proposed by Ahaan Rungta)