2014 International Physics Online Olympiad

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by jgf1123, ahaanomegas

– Open Olympiad, Round 1

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1 A capacitor is made with two square plates, each with side length L, of negligible thickness, and capacitance C. The two-plate capacitor is put in a microwave which increases the side length of each square plate by 1%. By what percent does the voltage between the two plates in the capacitor change?

(A) decreases by 2%
(B) decreases by 1%
(C) it does not change
(D) increases by 1%
(E) increases by 2%

Problem proposed by Ahaan Rungta

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2 An ice ballerina rotates at a constant angular velocity at one particular point. That is, she does not translationally move. Her arms are fully extended as she rotates. Her moment of inertia is I. Now, she pulls her arms in and her moment of inertia is now $\frac{7}{10}I$. What is the ratio of the new kinetic energy (arms in) to the initial kinetic energy (arms out)?

(A) $\frac{7}{10}$ **(B)** $\frac{49}{100}$ **(C)** 1 **(C)** $\frac{100}{49}$ **(E)** $\frac{10}{7}$

Problem proposed by Ahaan Rungta

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3 Which of the following derived units is equivalent to units of velocity?

(A) $\frac{\text{W}}{\text{N}}$ **(B)** $\frac{\text{N}}{\text{W}}$ **(C)** $\frac{\text{W}}{\text{N}^2}$ **(D)** $\frac{\text{W}^2}{\text{N}}$ **(E)** $\frac{\text{N}^2}{\text{W}^2}$

Problem proposed by Ahaan Rungta

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4 A rock is dropped off a cliff of height h As it falls, a camera takes several photographs, at random intervals. At each picture, I measure the distance the rock has fallen. Let the average (expected value) of all of these distances be kh. If the number of photographs taken is huge, find k. That is: what is the time-average of the distance traveled divided by h, dividing by h?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1}{\sqrt{3}}$

Problem proposed by Ahaan Rungta

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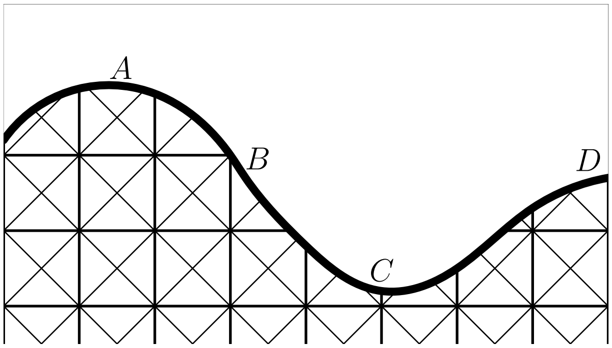
5 A frictionless roller coaster ride is given a certain velocity at the start of the ride. At which point in the diagram is the velocity of the cart the greatest? Assume a frictionless surface.



(A) A **(B)** B **(C)** C **(D)** D
(E) There is insufficient information to decide

Problem proposed by Kimberly Geddes

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6 A square plate has side length L and negligible thickness. It is laid down horizontally on a table and is then rotating about the axis $\overline{MN}$ where M and N are the midpoints of two adjacent sides of the square. The moment of inertia of the plate about this axis is kmL^2, where m is the mass of the plate and k is a real constant. Find k.

Diagram will be added to this post very soon. If you want to look at it temporarily, see the PDF.

Problem proposed by Ahaan Rungta

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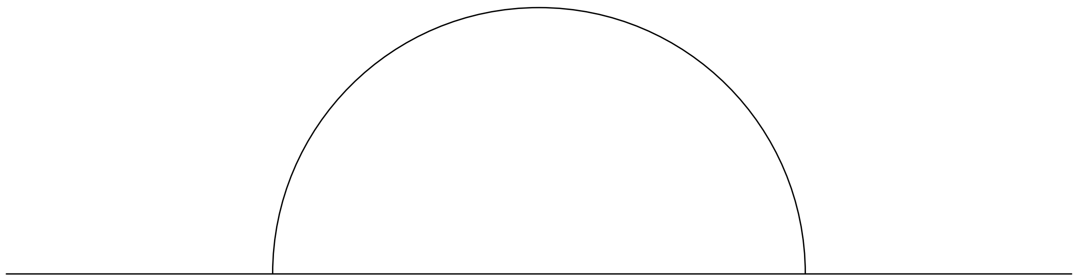
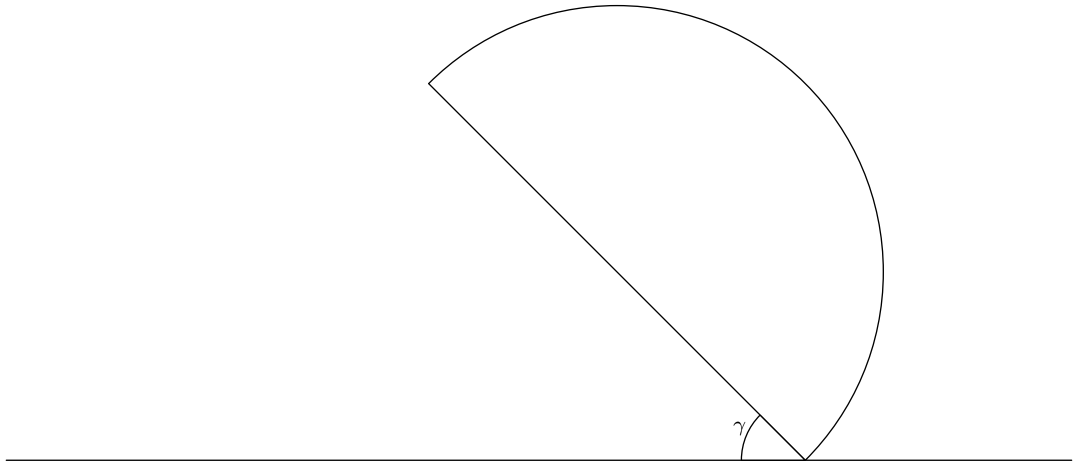
7 A uniform solid semi-circular disk of radius R and negligible thickness rests on its diameter as shown. It is then tipped over by some angle γ with respect to the table. At what minimum angle γ will the disk lose balance and tumble over? Express your answer in degrees, rounded to the nearest integer.





Problem proposed by Ahaan Rungta

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8 A plane, flying at a height of 3000 meters above the level ground below, receives a signal from the airport where the pilot intends to land. Using a vertical dipole antenna, the airport's air traffic control system is capable of transmitting 110-watt, 24 MHz signals. When the plane's horizontal position is 5 kilometers from the airport, what is the intensity of the signal at the plane's receiving antenna, in W/m^2? (The height of the transmitting antenna is negligible.)

Problem proposed by Kimberly Geddes

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9 An engineer is designing an engine. Each cycle, it ignites a negligible amount of fuel, releasing 2000 J of energy into the cubic decimeter of air, which we assume here is gaseous nitrogen at 20° C at 1 atm in the engine in a process which we can regard as instantaneous and isochoric. It then expands adiabatically until its pressure once again reaches 1 atm, and shrinks isobarically until it reaches its initial state. What is the efficiency of this engine?

Problem proposed by B. Dejean

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10 An electric field varies according the the relationship,

$$\mathbf{E} = \left(0.57\,\frac{\mathsf{N}}{\mathsf{C}}\right) \cdot \sin\left[\left(1720\,\mathsf{s}^{-1}\right) \cdot t\right].$$

Find the maximum displacement current through a $1.0\,\mathsf{m}^2$ area perpendicular to $\vec{\mathbf{E}}$. Assume the permittivity of free space to be $8.85 \times 10^{-12}\,\mathsf{F/m}$. Round to two significant figures.

Problem proposed by Kimberly Geddes

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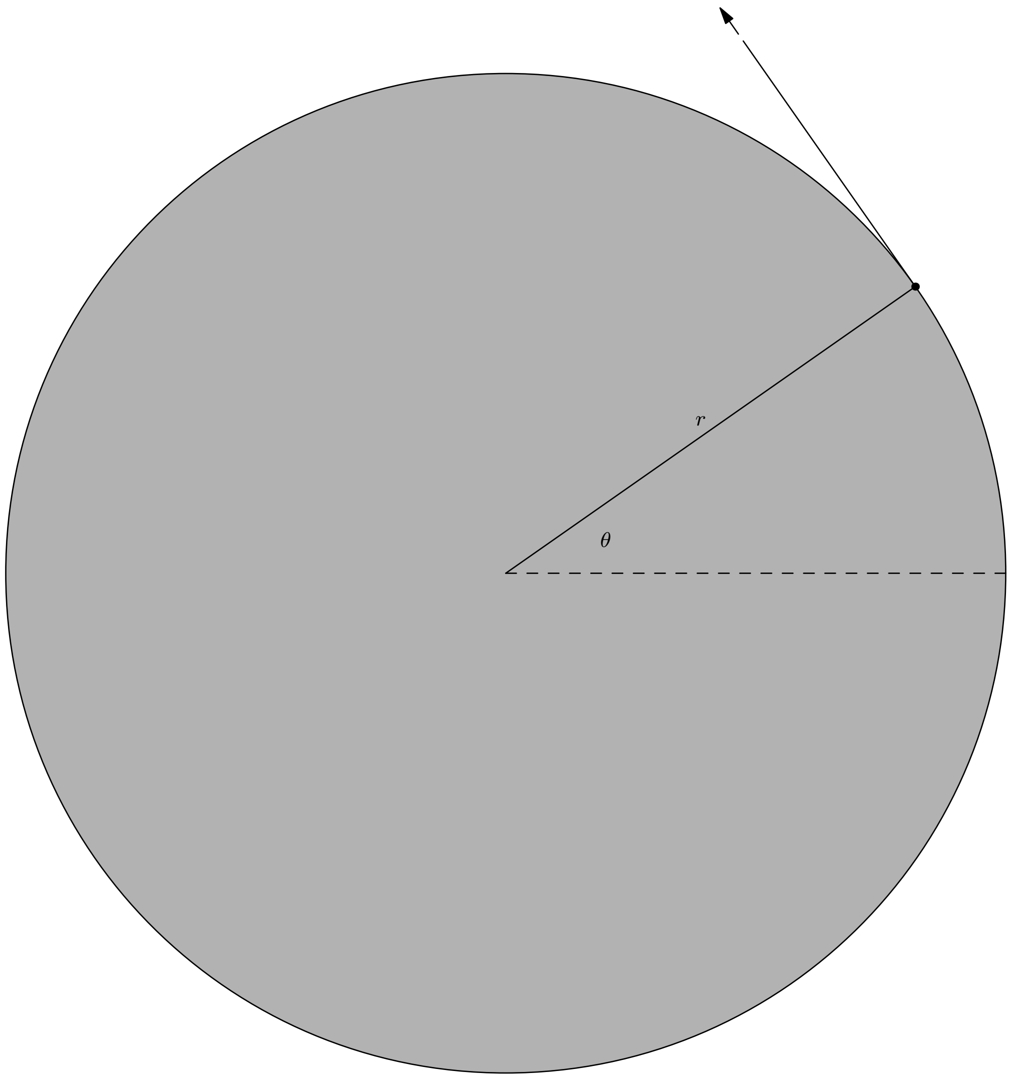
11 A spinning turntable is rotating in a vertical plane with period $500\,\mathsf{ms}$. It has diameter 2 feet carries a ping-pong ball at the edge of its circumference. The ball is bolted on to the turntable but is released from its clutch at a moment in time when the ball makes a subtended angle of $\theta > 0$ with the respect to the horizontal axis that crosses the center. This is illustrated in the figure. The ball flies up in the air, making a parabola and, when it comes back down, it does not hit the turntable. This can happen only if $\theta > \theta_m$. Find θ_m, rounded to the nearest integer degree?



Problem proposed by Ahaan Rungta

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12 A circular loop with radius r spins with angular frequency ω in a generated magnetic field of strength B. It is hooked to a resistor load R. How much work is done by the generator in one revolution?

Problem proposed by Ahaan Rungta

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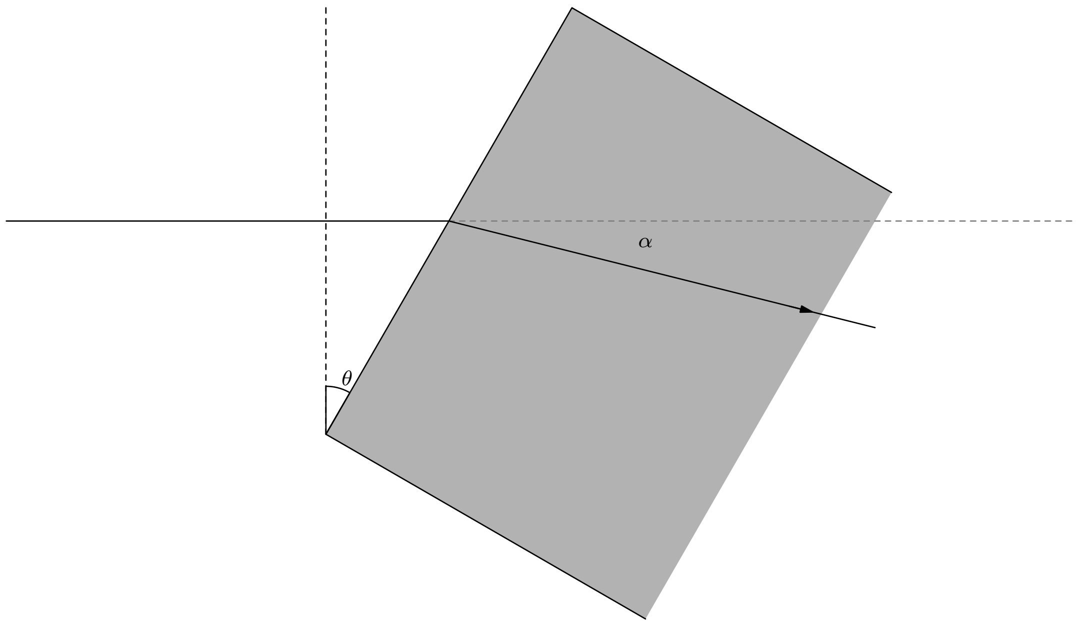
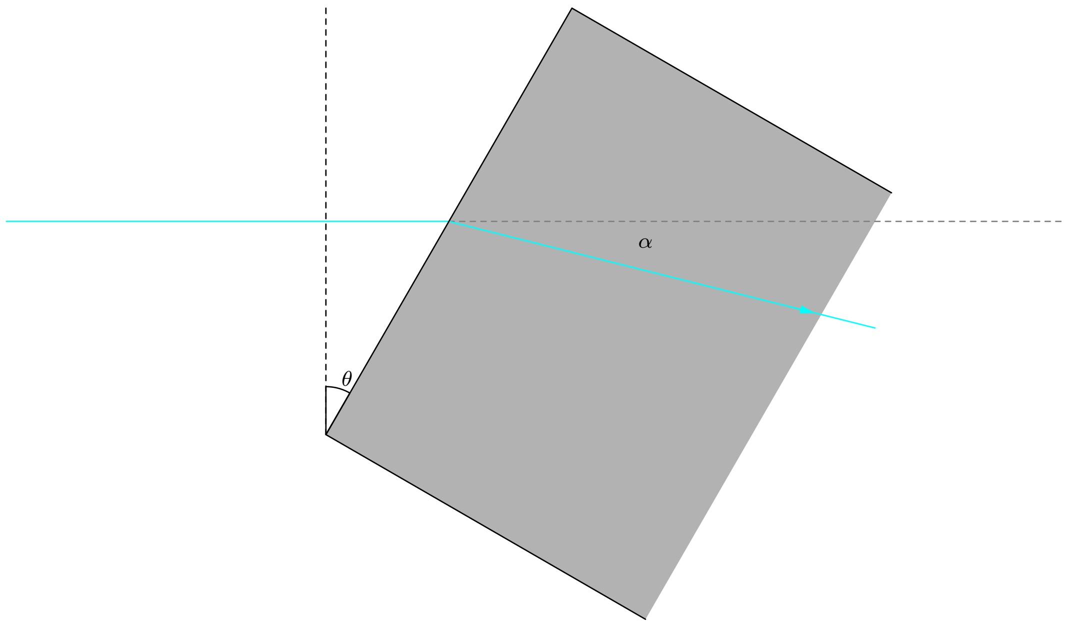
13 An infinitely long slab of glass is rotated. A light ray is pointed at the slab such that the ray is kept horizontal. If θ is the angle the slab makes with the vertical axis, then θ is changing as per the function

$$\theta(t) = t^2,$$

where θ is in radians. Let the *glassious ray* be the ray that represents the path of the refracted light in the glass, as shown in the figure. Let α be the angle the glassious ray makes with the horizontal. When $\theta = 30^\circ$, what is the rate of change of α, with respect to time? Express your answer in radians per second (rad/s) to 3 significant figures. Assume the index of refraction of glass to be 1.50. Note: the second figure shows the incoming ray and the glassious ray in cyan.





Problem proposed by Ahaan Rungta

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14 A super ball rolling on the floor enters a half circular track (radius R). The ball rolls without slipping around the track and leaves (velocity v) traveling horizontally in the opposite direction. Afterwards, it bounces on the floor. How far (horizontally) from the end of the track will the ball bounce for the second time? The balls surface has a theoretically infinite coefficient of static friction. It is a perfect sphere of uniform density. All collisions with the ground are perfectly elastic and theoretically instantaneous. Variations could involve the initial velocity being given before the ball enters the track or state that the normal force between the ball and the track right before leaving is zero (centripetal acceleration).

Problem proposed by Brian Yue

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15 The period of a given pendulum on a planet of radius R is constant (unchanged) as we go from the surface of the planet down to radius a, where $R > a$. The planet has mass density evenly distributed at any radius $r < a$. This density is ρ_0. Find the total mass of the planet. Express your answer in terms of ρ_0, a, R, the period of the pendulum, T, the length of the pendulum string, L, and other constants, as necessary.

Warning: Your answer may contain some math. So be sure to input this correctly!

Problem proposed by Trung Phan

– Round 2, Day 1

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1 The evil Dr. Doom seeks to destroy his enemy, the Intergalactic Federation, and has devised a plan to despin the Federation's space station. The hoop-shaped space station of mass M and radius R rotates once every T hours to maintain artificial gravity equal to that on IPhOO. Dr. Doom plans to mount two thruster rockets, one rocket on opposite sides of the space station, to stop its rotation. Dr. Doom must accomplish his crime within a time t to avoid getting caught. How much force should each rocket deliver in order to despin the Federation's space station in t? Express your answer in terms of M, R, T, t, and/or constants, as necessary.

Problem proposed by Kimberly Geddes

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2 Odysseus on his ten year return voyage to Ithaca sailed between two monsters. On one side, the creature Charybdis periodically sucked the oceans such that a whirlpool formed. On the opposing side the creature Scylla would lunge down from above and devour one sailor in each of her many mouths. Odysseus opted to sail near Scylla skirting Charybdis by 500 m. At this distance, the maximum drop in water level of the ocean was 0.2 m from between when Charybdis was draining the oceans and when he was not. At Charybdis' mouth the funnel of the whirlpool is 25 m wide. Assume that the oceans are perfectly calm and that there are no intermolecular attractions between water molecules.

(a) How deep is Charybdis under water?
(b) The boat with a crew of 40 men weighs 5,000 kg. Each crew member displaces 3 kg of water at a velocity of 5 m/s every stroke every second. If at some point Odysseus is traveling radially away from Charybdis, what is the closest his ship can be without being sucked in? Assume that Odysseus' vessel has an extremely shallow draft (low friction).

Problem proposed by Brian Yue

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3 Consider a very simple model for an open self-replicative system such as a cell, or an economy. A system $\mathbf{S}$ is comprised of two kinds of mass: one kind is $\mathbf{S}_R$ that is capable of of taking raw material that comes from outside the system, and converting it into components of the system, and the other is $\mathbf{S}_O$ that takes care of other things. Think of $\mathbf{S}_R$ like factories and $\mathbf{S}_O$ like street sweepers. One part is creating new things and the other is doing maintenance on what already exists. The catch is, the material in $\mathbf{S}_R$ must not only make the rest of the system, but also itself! Suppose that the materials in $\mathbf{S}_R$ and the materials in $\mathbf{S}_O$ cost the same amount of energy for $\mathbf{S}_R$ to make per unit amount. Suppose the material in $\mathbf{S}_R$ can convert raw material from the environment into system mass at the rate $\gamma_R = 3$ kg $\mathbf{S}$/hr/kg $\mathbf{S}_R$. If the system doubles in size once every 2 hrs, what fraction of the material in $\mathbf{S}$ is devoted to $\mathbf{S}_O$?

Assumptions: The fact that the system continuously doubles in size in a fixed time means that this system is in exponential growth, i.e. $\dot{\mathbf{S}} = \lambda \mathbf{S}$.

Problem proposed by Josh Silverman

– Round 2, Day 2

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1 A ring is of the shape of a hoola-hoop of negligible thickness. A ring of radius R carries a current I. Prove that the magnetic field at a given point in the plane of the ring at a distance a from the center, due to the magnetic field of the ring, is

$$B = \frac{\mu_0}{2\pi} \cdot IR \cdot \int_0^\pi \frac{R - a\cos\theta}{\sqrt{(a^2 + R^2 - 2aR\cos\theta)^3}} \,\mathrm{d}\theta.$$

Problem proposed by Ahaan Rungta

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2 An object has the shape of a square and has side length a. Light beams are shone on the object from a big machine. If m is the mass of the object, P is the power *per unit area* of the photons, c is the speed of light, and g is the acceleration of gravity, prove that the minimum value of P such that the bar levitates due to the light beams is

$$P = \frac{4cmg}{5a^2}.$$

Problem proposed by Trung Phan

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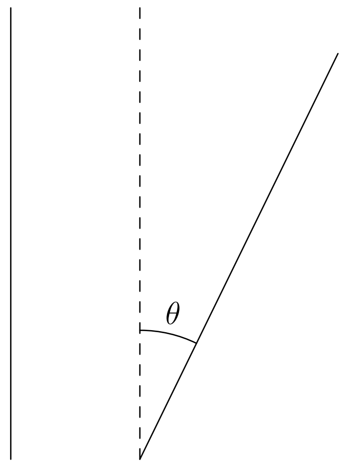
3 Consider a charged capacitor made with two square plates of side length L, uniformly charged, and separated by a very small distance d. The EMF across the capacitor is ξ. One of the plates is now rotated by a very small angle θ to the original axis of the capacitor. Find an expression for the difference in charge between the two plates of the capacitor, in terms of (if necessary) d, θ, ξ, and L.

Also, approximate your expression by transforming it to algebraic form: i.e. without any non-algebraic functions. For example, logarithms and trigonometric functions are considered non-algebraic. Assume $d << L$ and $\theta \approx 0$.

Hint: You may assume that $\frac{\theta L}{d}$ is also very small.

Problem proposed by Trung Phan

There are two possible ways to rotate the capacitor. Both were equally scored but this is what was meant:


θ